Decentralized Pricing*

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Abstract

This paper studies the conditions under which an intermediary can decentralize pricing decisions to privately informed parties of a transaction. Assuming first one-dimensional signals and negatively interdependent values, the paper shows that decentralized pricing is both necessary and sufficient for weak ex post implementation. Without negative interdependency, this result fails. Furthermore, considering arbitrary signal spaces, the paper shows a similar result for strong ex post implementation, provided that the values satisfy a separability assumption. Finally, optimal price mechanisms are considered.

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1 Introduction

When people seek to trade with one another, they typically make price offers, instead of plainly revealing privately known preferences and other information they may have about the object of trade. Indeed, direct communication of such information can be difficult, whereas quoting a price is straightforward. This begs the question: when do prices capture all the relevant information? For example, the potential buyer of a used car may ask the seller if the car has been in an accident or not. Yet maybe that information is already contained in the price asked by the seller, given that prospective buyers will care about such information.

To study this question, this paper considers the problem of an intemediary whose aim is to facilitate a transaction between two parties. If the parties complete the transaction, each of them obtains a value, positive or negative, which depends on information privately held by both parties. That is, each party observes a private signal about the value of the transaction, unknown to the other party and the intermediary. The intermediary seeks to implement an allocation, by choosing a game form, consisting of message spaces for the parties and of a decision rule. Following the economic literature on implementation under informational externalities, the solution concept of the game is ex post equilibrium in pure strategies.[1]

The paper shows that prices are sufficiently informative when informational externalities are negative. For instance, bad news about the quality of a good reduces the seller's opportunity cost of trade, but also makes buyers less willing to buy. When instead the values are positively interdependent, as is often the case in matching markets, for example, then prices do not suffice to capture all the relevant information.

Assuming one-dimensional signals and negatively interdependent values, the first

[1] A strategy profile of an incomplete information game is an ex post equilibrium if every action profile is a Nash equilibrium for every possible state of information (Bergemann and Morris, 2008).

result of the paper is that an allocation is weakly ex post implementable if and only if it can be weakly ex post implemented by a price mechanaism, in which each party sets a price that the other must pay to complete the transaction, and the intermediary makes a decision based on these prices.[2] If the transaction takes place, the intermediary charges the total price as a commission fee to the parties. Furthermore, independently of the transaction, each party may pay a fixed fee that depends on the price chosen by the other party.[3]

The intuition for this result is quite simple. Returning to the bilateral trade example, if a seller with good news ends up selling the good, then sellers with bad news will also sell. Having lower opportunity costs from trade, they benefit from the transaction more and can always mimic the seller with good news. To provide the correct incentives for a buyer to purchase under bad news, the price must then decrease, as the buyer benefits less from the transaction. It then follows that, for every price charged to the buyer, there exists a unique signal observed by the seller, leading to our result by the Revelation Principle.[4] Without negative interdependency this result fails, because two different signals can be associated with the same price, but still lead to different allocations.

Furthermore, considering arbitrary signal spaces, the paper shows that decentralized pricing is necessary and sufficient for strong ex post implementation, provided that the values are negatively interdependent and satisfy a separability assumption.[5] Under these two assumptions, the allocation is strongly ex post implementable if and only if there exists an *augmented* price mechanism with a unique ex post equilibrium, where

[2]The allocation is weakly ex post implementable if there exists a game form for which there exists an ex post equilibrium of the game that delivers the allocation. This performance standard permits other ex post equilibria, which do not yield the allocation.

[3]Without individual rationality constraints these fees are arbitrary.

[4]According to this principle, the allocation is weakly ex post implementable if and only if truth-telling constitutes an ex post equilibrium of the direct mechanism, where each party is asked to report privately held information directly.

[5]The allocation is strongly ex post implementable if it is weakly ex post implementable and every ex post equilibria of the game delivers the allocation.

each party bids the equilibrium transaction price charged to the other party.[6] If two different types choose the same transaction price, strong ex post implementability requires that they have the same allocation. Otherwise there exists an ex post equilibrium of the game, where one of the types mimics the other type, implementing a different allocation.

As a solution concept, ex post equilibrium refines Bayesian equilibrium with the additional property of no regret: once the resources have been allocated, no player of the incomplete information game would like to change action even if private information were to become public.[7] However, ex post equilibrium strategies are not necessarily dominant strategies; they need not to best respond to out of equilibrium actions. Thus, for weak implementation, ex post equilibrium is a stronger concept than Bayesian equilibrium, but weaker than equilibrium in dominant strategies. For strong implementation this ranking is no longer true, as the performance standard requires that every equilibria of the game result in the desired allocation.

Related literature It is standard to study the implications of informational externalities in ex post equilibrium. As opposed to other Bayesian equilibria, ex post equilibria are robust to assumptions about the informational structure, so the results hold no matter what beliefs players have about each other. Bergemann and Morris (2005) show that ex post implementation is equivalent to Bayesian implementation in separable environments. Furthermore, when informational externalities are present, ex post equilibrium is much more tractable than dominant strategies, which allow to im-

[6]When a price mechanism is augmented, the parties may also choose other messages than prices, but these arbitrary messages are never played in equilibrium. This idea is linked to Mookherjee and Reichelstein (1990) who study implementation via augmented direct mechanisms.

[7]Hence the name of the concept. Holmström and Myerson (1983) define three stages, depending on how much information is available to the players: the ex ante stage, before the players have observed any private information; the interim stage, when the players have learnt their own private information, but not what the others have observed; the ex post stage, when all information is public knowledge.

plement very little if nothing at all (Williams and Radner, 1988). But when there are no informational externalities, weak implementation in dominant strategies is equivalent to weak ex post implementation. Bergemann and Morris (2008) provide necessary and sufficient conditions for strong ex post implementation.

Most economic literature on informational externalities considers one-dimensional signals. Milgrom and Weber (1982) introduce a model of auctions with informational externalities and Crémer and McLean (1985) provide sufficient conditions for extracting the entire buyer surplus in ex post equilibrium when demands are interdependent. Dasgupta and Maskin (2000) and Motty and Philip (2002) study ex post efficient auctions, whereas Birulin (2003) shows inefficient ex post equilibria in them. Dasgupta and Maskin (2000) show that, when signals are multi-dimensional, no auction is generally efficient. Jehiel et al. (2006) study the limits of ex post implementation and consider more general frameworks with multi-dimensional signals. They show that the only deterministic allocations that are generically ex post implementable, are constant functions. A similar result is obtained by Jehiel and Moldovanu (2001) on Bayesian implementation.

The paper also relates to the literature on bilateral trade. Myerson and Satterthwaite (1983) show the general impossibility of ex post efficient and individually rational, Bayesian incentive compatible trading mechanisms. Chatterjee and Samuelson (1983) consider bilateral bargaining without an intermediary: under a Nash bargaining rule, the buyer and the seller simultaneously submit price offers, which determine whether the good is sold and at what price. Hagerty and Rogerson (1987) consider the problem of designing an efficient and ex post budget-balanced trading institution in dominant strategies. They show that posted-price mechanisms are essentially the only mechanisms in which each trader has a dominant strategy.

The paper is organized as follows. The next section introduces the model and defines price mechanisms. Section 3 is devoted to one-dimensional signal spaces and shows the main result of the paper, focusing on weak ex post implementation. Section 4 then derives the result on strong ex post implementation, which holds for arbitrary signal spaces. Section 5 studies optimal price mechanisms, including the efficient one and the one that maximizes the expected revenue from intermediation, subject to a prior over the signal spaces.

2 Model

An intermediary brings together two parties. If the parties complete a transaction, each party $i \in \{1, 2\}$ obtains a value $v_i(x) \in \mathbb{R}$, possibly negative, where $x = (x_1, x_2)$ denotes a type profile in $X = X_1 \times X_2$. The type or signal x_i is drawn from a nonempty space X_i and privately observed by party i. It is unknown to the other party j and the intermediary. The intermediary is not informed about the value of the transaction.

Both parties have additively separable utility in money and the transaction. If party i makes a monetary transfer $t_i \in \mathbb{R}$ and $q \in \{0, 1\}$ indicates whether the transaction takes place or not, the party obtains utility

$$U_i(q, t_i; x) = v_i(x)\, q - t_i.$$

The total payment $t_1 + t_2$ captures the revenue collected by the intermediary, seeking to implement an allocation by choosing a mechanism. We adopt the following definitions:

Definition 1. An allocation $a = (q, t)$ consists of functions $q : X \longrightarrow \{0, 1\}$ and $t = (t_1, t_2)$, where $t_i : X \longrightarrow \mathbb{R}$. For any $x \in X$, $q(x)$ indicates if the transaction takes place or not, and $t_i(x)$ is the monetary transfer made by party i.

Remark 1. The allocation is deterministic; there is no randomization even if both parties are indifferent between transacting or not.

Definition 2. A mechanism (M, g) is a message space $M = M_1 \times M_2$ and a decision rule $g = (Q, T)$, which consists of functions $Q : M \longrightarrow \{0, 1\}$ and $T = (T_1, T_2)$, where $T_i : M \longrightarrow \mathbb{R}$. For any $m \in M$, $Q(m)$ indicates if the transaction takes place or not, and $T_i(m)$ is the monetary transfer made by party i.

Combined with the signal space X, a mechanism (M, g) describes a game of incomplete information, where the intermediary first commits to the mechanism and then each party i of type $x_i \in X_i$ chooses a message $m_i \in M_i$. For any pair of messages m, the decision $g(m)$ determines the payoffs

$$U_i\left(Q(m), T_i(m); x\right) = v_i(x)\, Q(m) - T_i(m)$$

and the total payment $T_1(m) + T_2(m)$. In this game, a pure strategy of party i is a function $s_i : X_i \longrightarrow M_i$ and $s = (s_1, s_2)$ denotes a profile of strategies. The equilibrium concept is ex post equilibrium in pure strategies:

Definition 3. A strategy profile s^* constitutes an ex post equilibrium of the game if for each party $i \in \{1, 2\}$ and every $x \in X$:

$$s_i^*(x_i) \in \arg\max_{m_i \in M_i} U_i\left(Q\left(m_i, s_j^*(x_j)\right), T_i\left(m_i, s_j^*(x_j)\right); x\right).$$

In other words, the strategy profile s^* is an ex post equilibrium if and only if the action profile $(s_1^*(x_1), s_2^*(x_2))$ is a Nash equilibrium for every $x \in X$. Thus, an ex post equilibrium is a Bayesian equilibrium with no regret: even if the signal observed by party j were to become public, party i would have no incentive to change action.

We consider both weak and strong ex post implementation:

Definition 4. The allocation $a = (q, t)$ is *weakly* ex post implementable if there exists a mechanism (M, g) for which there exists an ex post equilibrium s^* of the game such that

$$a = g \circ s^*.$$

Definition 5. The allocation $a = (q, t)$ is *strongly* ex post implementable if it is weakly ex post implementable and there exists a mechanism (M, g) such that, for every ex post equilibrium s^* of the game,

$$a = g \circ s^*.$$

By the Revelation Principle, the allocation a is weakly ex post implementable if and only if truth-telling constitutes an ex post equilibrium of the direct mechanism (X, a).

Two-part tariffs Applying the Revelation Principle, for arbitrary signal spaces, we obtain the following characterization of weakly ex post implementable allocations:

Lemma 1. *The allocation $a = (q, t)$ is weakly ex post implementable if and only if for each $i \in \{1, 2\}$ there exist functions:*

$$f_i : X_j \longrightarrow \mathbb{R},$$

$$p_i : X_j \longrightarrow \mathbb{R},$$

such that, for every $x \in X$, the payment to the intermediary can be written as a two-part tariff

$$t_i(x_i, x_j) = \overbrace{f_i(x_j)}^{\textit{fixed fee}} + \overbrace{p_i(x_j)}^{\textit{price}} \cdot q(x_i, x_j),$$

where

$$q(x_i, x_j) = \begin{cases} 1 & \textit{if } v_i(x_i, x_j) > p_i(x_j), \\ 0 & \textit{if } v_i(x_i, x_j) < p_i(x_j), \end{cases}$$

and

$$p_i(x_j) \in [\underline{v}_i(x_j) := \inf v_i(X_i, x_j), \overline{v}_i(x_j) := \sup v_i(X_i, x_j)].$$

Proof. Step 1: $\Longrightarrow$). Let us first show that the two-part tariff structure implies that the allocation $a = (q, t)$ is weakly ex post implementable by the direct mechanism (X, a). For every $i \in \{1, 2\}$, $x \in X$ and $m_i \in M_i = X_i$:

$$\begin{aligned} U_i(q(m_i, x_j), t_i(m_i, x_j); x) &= v_i(x_i, x_j) q(m_i, x_j) - t_i(m_i, x_j) \\ &= [v_i(x_i, x_j) - p_i(x_j)] q(m_i, x_j) - f_i(x_j) \\ &\leq [v_i(x_i, x_j) - p_i(x_j)] q(x_i, x_j) - f_i(x_j) \\ &= U_i(q(x_i, x_j), t_i(x_i, x_j); x), \end{aligned}$$

where the inequality follows from the fact that

$$\begin{aligned} v_i(m_i, x_j) > p_i(x_j) &\Longrightarrow q(m_i, x_j) = 1, \\ v_i(m_i, x_j) < p_i(x_j) &\Longrightarrow q(m_i, x_j) = 0. \end{aligned}$$

Thus, telling the truth constitutes an ex post equilibrium of the direct mechanism.

Step 2: $\Longleftarrow$). Let us now show that the two-part tariff structure is necessary for weak ex post implementability. Assume a is weakly ex post implementable. Then, by the Revelation Principle, truth-telling constitutes an equilibrium of the direct mechanism (X, a). Consider party $i \in \{1, 2\}$. Fix any $x_i, x_i' \in X_i$ and any $x_j \in X_j$ for $j \neq i \in \{1, 2\}$.

By ex post incentive compatibility:

$$v_i(x_i, x_j) q(x_i, x_j) - t_i(x_i, x_j) \geq v_i(x_i, x_j) q(x_i', x_j) - t_i(x_i', x_j),$$
$$v_i(x_i', x_j) q(x_i', x_j) - t_i(x_i', x_j) \geq v_i(x_i', x_j) q(x_i, x_j) - t_i(x_i, x_j).$$

Combining these ex post incentive compatibility constraints yields

$$\begin{aligned} v_i(x_i', x_j)[q(x_i', x_j) - q(x_i, x_j)] &\geq t_i(x_i', x_j) - t_i(x_i, x_j) \\ &\geq v_i(x_i, x_j)[q(x_i', x_j) - q(x_i, x_j)]. \end{aligned}$$

Thus $v_i(x_i', x_j) > v_i(x_i, x_j)$ implies $q(x_i', x_j) \geq q(x_i, x_j)$ and $q(x_i', x_j) = q(x_i, x_j)$ implies $t_i(x_i', x_j) = t_i(x_i, x_j)$. Therefore, for every $x_i \in X_i$ such that $q(x_i, x_j) = \bar{q}$ we may define $t_i^{\bar{q}}(x_j) \coloneqq t_i(x_i, x_j)$. Furthermore, define

$$p_i(x_j) \coloneqq \begin{cases} \overline{v}_i(x_j) & \text{if } q(x_i, x_j) = 0, \forall x_i \in X_i, \\ \underline{v}_i(x_j) & \text{if } q(x_i, x_j) = 1, \forall x_i \in X_i, \\ t_i^1(x_j) - t_i^0(x_j) & \text{otherwise,} \end{cases}$$

where $\overline{v}_i(x_j) = \sup v_i(X_i, x_j)$ and $\underline{v}_i(x_j) = \inf v_i(X_i, x_j)$, and

$$f_i(x_j) \coloneqq t_i(x_i, x_j) - p_i(x_j) q(x_i, x_j).$$

Using these definitions, the ex post incentive compatibility constraints can be written as

$$\begin{aligned} v_i(x_i', x_j)[q(x_i', x_j) - q(x_i, x_j)] &\geq p_i(x_j)[q(x_i', x_j) - q(x_i, x_j)] \\ &\geq v_i(x_i, x_j)[q(x_i', x_j) - q(x_i, x_j)]. \end{aligned}$$

As $v_i(x_i', x_j) > v_i(x_i, x_j)$ implies $q(x_i', x_j) \geq q(x_i, x_j)$, by the definition of $p_i(x_j)$ we have

$$v_i(x_i, x_j) > p_i(x_j) \Longrightarrow q(x_i, x_j) = 1,$$
$$v_i(x_i, x_j) < p_i(x_j) \Longrightarrow q(x_i, x_j) = 0,$$

which imply

$$p_i(x_j) \in \left[\underline{v}_i(x_j), \overline{v}_i(x_j)\right].$$

□

Lemma 1 shows that a two-part tariff structure is both necessary and sufficient for weak ex post implementation. If the parties complete the transaction, depending on the signal observed by the other party j, party i pays a transaction price $p_i(x_j) \in \left[\underline{v}_i(x_j), \overline{v}_i(x_j)\right]$. The total transaction price captures the commission fee paid to the intermediary for executing the deal. Furthermore, independently of the transaction, each party i pays a fixed fee $f_i(x_j) \in \mathbb{R}$, which also depends on the signal observed by the other party j. The total fixed fee adds to the revenue from intermediation.

The intuition for Lemma 1 is the following. First, by incentive compatibility, the payments may only depend on the signal observed by the other party and whether the transaction occurs or not. Otherwise, there exists a type with an incentive to mimic another type with the same decision about the transaction but a lower payment. We may then express each payment in two parts: a fixed fee that is paid independently of the transaction, and a price that is paid only if the transaction takes place. One can interpret this argument as an application of the Taxation Principle.[8]

[8] See Salanié (2005) for instance.

Second, when a given type x_j of party j always completes the transaction, there is a degree of freedom in choosing the associated two-part tariff. We may choose $p_i(x_j) = \overline{v}_i(x_j)$ and adjust the fixed fee $f_i(x_j)$ accordingly. Likewise, if the type never completes the transaction, we may set $p_i(x_j) = \underline{v}_i(x_j)$ as the price is never paid. But then, given these choices, incentive compatibility requires that

$$v_i(x_1, x_2) > p_i(x_j) \Longrightarrow q(x_1, x_2) = 1,$$
$$v_i(x_1, x_2) < p_i(x_j) \Longrightarrow q(x_1, x_2) = 0,$$

and thus $p_i(x_j) \in [\underline{v}_i(x_j), \overline{v}_i(x_j)]$.

Price mechanisms The two-part tariff structure suggests a simple indirect mechanism, where instead of reporting their private information directly, each party sets a price, and the intermediary makes a decision based on these prices. Formally, price mechanisms are defined as follows:

Definition 6. A price mechanism (P, g) is a mechanism with message spaces $P_j \subseteq \mathbb{R}$ and a decision rule $g = (Q, T)$, such that, for each party $i \in \{1, 2\}$ and every $p \in P = P_2 \times P_1$ the monetary transfer can be written as

$$T_i(p) = F_i(p_i) + p_i Q(p),$$

where $F_i : P_i \longrightarrow \mathbb{R}$ determines the fixed fee.

The interpretation of a price mechanism (P, g) is a double auction, where *the other party* j chooses a price $p_i \in P_i$ party i needs to pay to complete the transaction:

$$U_i(Q(p), T_i(p); x) = (v_i(x) - p_i) Q(p) - F_i(p_i),$$

where the fixed fee $F_i(p_i)$ depends on the pricing decision. The total price $p_1 + p_2$ captures the commission fee paid to the intermediary for executing the deal. Accounting for the fixed fees, the intermediary has revenue

$$T_1(p) + T_2(p) = (p_1 + p_2) Q(p) + F_1(p_1) + F_2(p_2).$$

We are interested in implementation by price mechanisms, because they are particularly simple. Both parties are asked to quote a price instead of reporting their private information, which can be complex and difficult for the parties to communicate directly. We now turn to the main results of the paper, by first analyzing weak ex post implementation in a basic environment with one-dimensional signals and then looking at strong ex post implementation with arbitrary signal spaces. After this, we characterize optimal price mechanisms.

3 One-dimensional signals

Following current economic literature, it seems natural to begin by considering the basic environment, where private information held by each party can be summarized by a single number, drawn from an interval of real numbers. Hence, throughout this subsection, we shall assume $X_i = [\underline{x}_i, \overline{x}_i] \subseteq \mathbb{R}$ for both $i \in \{1, 2\}$. Furthermore, we make the following assumption:

Assumption 1. *For every $i \in \{1, 2\}$, $v_i(x)$ is continuous, strictly increasing in x_i and strictly decreasing in x_j.*

This means that the values are negatively interdependent: if party i observes a higher signal and thus values the transaction more, mutatis mutandis, the other party j values the transaction less. This assumption seems realistic in many contexts. In the case of a buyer and a seller, for example, this is equivalent to saying that they have positively correlated signals about the value of the object, owned by the seller: a high signal increases the value of the good for both parties, making the seller less willing to trade and the buyer more eager to purchase.

Together with the two-part tariff structure, which is necessary and sufficient for weak ex post implementation, negative interdependence of transaction values implies strictly decreasing transaction price functions:

Lemma 2. *Under Assumption 1, for each party $j \neq i \in \{1,2\}$, the price $p_i(x_j)$, characterized by Lemma 1, is strictly decreasing in x_j.*

Proof. Suppose, by contradiction, that there exist $x_i, x_i' \in X_i$ such that $x_i' > x_i$ and $p_j(x_i') \geq p_j(x_i)$. By continuity and Lemma 1, there exist $x_j, x_j' \in X_j$ such that

$$v_j\left(x_i', x_j'\right) = p_j(x_i') \geq p_j(x_i) = v_j(x_i, x_j). \tag{1}$$

By negative interdependency, $x_i' > x_i$ implies $v_j\left(x_i, x_j'\right) > v_j\left(x_i', x_j'\right)$ so that (1) implies $v_j\left(x_i, x_j'\right) > v_j(x_i, x_j)$, which in turn implies $x_j' > x_j$, as the values are strictly increasing in own signals. Furthermore, for any $\hat{x}_j \in \left(x_j, x_j'\right)$ we have $v_j(x_i', \hat{x}_j) < p_j(x_i')$ and $p_j(x_i) < v_j(x_i, \hat{x}_j)$. By Lemma 1, we then have:

$$\begin{aligned} v_j(x_i, \hat{x}_j) > p_j(x_i) &\Longrightarrow q(x_i, \hat{x}_j) = 1 \Longrightarrow v_i(x_i, \hat{x}_j) \geq p_i(\hat{x}_j), \\ v_j(x_i', \hat{x}_j) < p_j(x_i') &\Longrightarrow q(x_i', \hat{x}_j) = 0 \Longrightarrow v_i(x_i', \hat{x}_j) \leq p_i(\hat{x}_j). \end{aligned}$$

But then $v_i(x_i, \hat{x}_j) \geq v_i(x_i', \hat{x}_j)$, which implies $x_i \geq x_i'$, contradicting $x_i' > x_i$. Thus $p_j(x_i') < p_j(x_i)$ must hold. □

Lemma 2 uses negative interdependency to show that the transaction price functions must be strictly decreasing for the allocation to be weakly ex post implementable.

Intuitively, if a low type completes the transaction, incentive compatibility requires that all higher types will also trade, because they value the transaction more and would otherwise mimic the low type. However, under negative interdependency, the other party benefits less from a transaction with a higher type, implying that the price must decrease to provide the correct incentives.

Remark 2. Strict monotonicity of the price functions implies that the intermediary must charge a lower total price to parties who observe high signals about the value of the transaction. In equilibrium, a party with a high signal sets a lower price to the other party, which highlights the cost of providing the correct incentives for the parties.

As prices are strictly decreasing, for every price paid by one of the parties, there exists a unique signal observed by the other party. Such injectivity yields a result on weak ex post implementability:

Proposition 1. *Under Assumption A, the allocation $a = (q, t)$ is weakly ex post implementable if and only if there exists a price mechanism (P, g) that weakly ex post implements it.*

Proof. Assume the allocation $a = (q, t)$ is weakly ex post implementable. Then, by Lemma 1, the allocation satisfies the two-part tariff structure. Furthermore, by Lemma 2, both price functions are strictly decreasing. Thus, there exists a price mechanism such that for each $i \in \{1, 2\}$: $P_i = p_i(X_j)$ and for every $p = (p_2, p_1) \in P$:

$$Q(p) = q\left(p_2^{-1}\{p_2\}, p_1^{-1}\{p_1\}\right),$$
$$F_i(p_i) = f_i\left(p_i^{-1}\{p_i\}\right).$$

But then, since $a = (q, t)$ is weakly ex post implementable, by the Revelation Principle, truth-telling is an ex post equilibrium of the direct mechanism, which is equivalent to saying that $s^* = (p_2, p_1)$ constitutes an ex post equilibrium of the price mechanism. $\square$

On the contrary, when the values are not negatively interdependent, there exist weakly ex post implementable allocations that cannot be implemented by any price mechanism: the price functions may have flat parts so that two different types, who are associated with the same transaction price, but face different decisions about the transaction:

Proposition 2. *Suppose the values are not negatively interdependent. Then, there exists a weakly ex post implementable allocation that cannot be weakly ex post implemented by any price mechanism.*

Proof. If the values are not negatively interdependent, there exist $x_i, x_i' \in X_i$ such that $x_i' > x_i$ and $v_j(x_i', x_j) \geq v_j(x_i, x_j)$ for some $x_j \in X_j$. By Lemma 1 we may then construct a weakly ex post implementable allocation $a = (q, t)$ that satisfies $q(x_i, x_j) = 0$, $q(x_i', x_j) = 1$ and $p_j(x_i') = p_j(x_i)$. Clearly no price mechanism implements this allocation. □

4 Arbitrary signals

Let us now relax the assumption of one-dimensionality and consider arbitrary signal spaces. Indeed, the parties may have private information on several aspects, which is exactly the reason why a price mechanism is convenient, as it only requires each party to tell a number. The assumption of negatively interdependent values can be stated more generally as follows:

Assumption 2. *For every $j \neq i \in \{1, 2\}$, every $x_i, x_i' \in X_i$ and every $x_j \in X_j$:*

$$v_i(x_i', x_j) > v_i(x_i, x_j) \Longrightarrow v_j(x_i', x_j) < v_j(x_i, x_j).$$

We will also need a separability assumption:

Assumption 3. *There exist functions* $v_{ii} : X_i \longrightarrow \mathbb{R}$ *and* $v_{ij} : X_j \longrightarrow \mathbb{R}$ *such that, for every* $j \neq i \in \{1, 2\}$ *and* $x \in X$*, we can write*

$$v_i (x_i, x_j) = v_{ii} (x_i) + v_{ij} (x_j) .$$

Under these two assumptions, which we keep throughout this section, we can show the following requirement for strong ex post implementation:

Lemma 3. *The allocation* $a = (q, t)$ *is strongly ex post implementable only if for each party* $i \in \{1, 2\}$ *and every* $x, x' \in X$*:*

$$p_i \left(x'_j\right) = p_i (x_j) \Longrightarrow q \left(x_i, x'_j\right) = q (x_i, x_j) \ \text{ and } f_i \left(x'_j\right) = f_i (x_j) ,$$

where $\{p_i, k_i\}$ *are characterized by Lemma 1.*

Proof. Assume the allocation $a = (q, t)$ is strongly ex post implementable. Then, by definition, there exists a mechanism (M, g) such that, for every ex post equilibrium s^* of the game: $a = g \circ s^*$. Furthermore, ex post equilibria exist. As weak ex post implementation is necessary for strong ex post implementation, a can be written as two-part tariffs according to Lemma 1. Fix any $x_i, x'_i \in X_i$ such that $p_j (x'_i) = p_j (x_i)$. Suppose, by contradiction, that

$$\hat{X}_j = \{x_j \in X_j : q (x'_i, x_j) \neq q (x_i, x_j)\}$$

is nonempty and, without loss, consider any $\hat{x}_j \in \hat{X}_j$ such that $q (x'_i, \hat{x}_j) = 1$ and

$q(x_i, \hat{x}_j) = 0$. Then, by Lemma 1:

$$q(x_i, \hat{x}_j) = 0 \Longrightarrow v_j(x_i, \hat{x}_j) \leq p_j(x_i) \text{ and } p_i(\hat{x}_j) \leq v_i(x_i, \hat{x}_j),$$
$$q(x_i', \hat{x}_j) = 1 \Longrightarrow v_j(x_i', \hat{x}_j) \geq p_j(x_i') \text{ and } p_i(\hat{x}_j) \geq v_i(x_i', \hat{x}_j).$$

Using the hypothesis $p_j(x_i') = p_j(x_i)$ we have

$$v_j(x_i', \hat{x}_j) \geq p_j(x_i') = p_j(x_i) \geq v_j(x_i, \hat{x}_j)$$

and

$$v_i(x_i, \hat{x}_j) \geq p_i(\hat{x}_j) \geq v_i(x_i', \hat{x}_j),$$

which by Assumption 2 imply that

$$v_j(x_i', \hat{x}_j) = v_j(x_i, \hat{x}_j),$$
$$v_i(x_i, \hat{x}_j) = v_i(x_i', \hat{x}_j).$$

Thus, for every $x_j \in X_j$:

$$x_j \in \hat{X}_j \Longrightarrow v_j(x_i', x_j) = p_j(x_i') = p_j(x_i) = v_j(x_i', x_j) \text{ and}$$
$$v_j(x_i, x_j) = p_i(x_j) = v_i(x_i', x_j),$$
$$x_j \notin \hat{X}_j \Longrightarrow q(x_i, x_j) = q(x_i', x_j),$$

where the latter implication is by definition of $\hat{X}_j$. But then, for every $x_j \in X_j$:

$$\begin{aligned} U_i\left(a\left(x_i', x_j\right); x_i, x_j\right) &= \left[p_i\left(x_j\right) - v_i\left(x_i, x_j\right)\right] q\left(x_i', x_j\right) + f_i\left(x_j\right) \\ &= \left[p_i\left(x_j\right) - v_i\left(x_i, x_j\right)\right] q\left(x_i, x_j\right) + f_i\left(x_j\right) \\ &= U_i\left(a\left(x_i, x_j\right); x_i, x_j\right). \end{aligned}$$

Using this equality, we can show that, if party j plays the equilibrium strategy, party i has a best response to mimic x_i' at true type x_i. Indeed, for any ex post equilibrium s^* in (M, g) we have that for any $m_i \in M_i$ and $x_j \in X_j$:

$$\begin{aligned} U_i\left(g\left(s_i^*\left(x_i'\right), s_j^*\left(x_j\right)\right); x_i, x_j\right) &= U_i\left(a\left(x_i', x_j\right); x_i, x_j\right) \\ &= U_i\left(a\left(x_i, x_j\right); x_i, x_j\right) \\ &= U_i\left(g\left(s_i^*\left(x_i\right), s_j^*\left(x_j\right)\right); x_i, x_j\right) \\ &\geq U_i\left(g\left(m_i, s_j^*\left(x_j\right)\right); x_i, x_j\right). \end{aligned}$$

As $\hat{X}_j$ is not empty, by strong ex post implementability, party i mimicing x_i' at true type x_i does not constitute an ex post equilibrium. Therefore, there exist $x_j \in X_j$ and $m_j \in M_j$ such that party j will deviate:

$$U_j\left(g\left(s_i^*\left(x_i'\right), s_j^*\left(x_j\right)\right); x_i, x_j\right) < U_j\left(g\left(s_i^*\left(x_i'\right), m_j\right); x_i, x_j\right).$$

Furthermore, by the definition of an ex post equilibrium:

$$U_j\left(g\left(s_i^*\left(x_i'\right), s_j^*\left(x_j\right)\right); x_i', x_j\right) \geq U_j\left(g\left(s_i^*\left(x_i'\right), m_j\right); x_i', x_j\right).$$

Together these two inequalities imply $v_j\left(x_i, x_j'\right) \neq v_j\left(x_i', x_j'\right)$. But then, by the separability assumption, $v_j\left(x_i, x_j\right) \neq v_j\left(x_i', x_j\right)$ must be true for all $x_j \in X_j$, implying that $\hat{X}_j$ is empty, a contradiction. Thus, $p_j(x_i') = p_j(x_i)$ implies that, for all $x_j \in X_j$ we have $q(x_i', x_j) = q(x_i, x_j)$. This in turn implies $f_j(x_i') = f_j(x_i)$ by strong ex post implementability. □

The idea for the proof of Lemma 3 is the following. If two types associated with the same transaction price have different decisions about the transaction for some type of the other party, then under negative interdependency incentive compatibility implies that both types must be indifferent between completing the transaction or not. But then, strong ex post implementability requires that the other party will deviate from the equilibrium if the types mimic each other, without upsetting the truth-telling equilibrium. This then contradicts the separability assumption.

Relationship to direct mechanisms By Lemma 3, for any weakly ex post implementable allocation, we may define a price mechanism (P, g) with the property $a = g \circ p$, where p denotes the pair of transaction price functions. This directly gives us the following result on equivalence between direct implementation and implementation by price mechanisms:

Proposition 3. *The allocation $a = (q, t)$ is strongly ex post implementable by the direct mechanism (X, a) if and only if there exists a price mechanism (P, g) with a unique ex post equilibrium*

$$s^* = (s_1^*, s_2^*) = (p_2, p_1),$$

strongly ex post implementing the allocation.

Proof. Follows immediately from Lemma 3. For uniqueness, note that strong ex post implementability requires that the price mechanism has no other ex post equilibria, because, by definition, those would involve different transaction prices. □

However, Proposition 3 permits the existence of an allocation, which is strongly ex post implementable an indirect mechanism, but not necessarily strongly ex post implementable by the direct mechanism nor a price mechanism.

Augmented price mechanisms Mookherjee and Reichelstein (1990) solve this issue by considering augmented direct mechanisms, by augmenting the type spaces with arbitrary messages that are never played in equilibrium. In a similar fashion, let us consider augmented price mechanisms:

Definition 7. An augmented price mechanism $(P \cup A, g)$ is a mechanism with message spaces $P_j \cup A_i$, where $P_j \subseteq \mathbb{R}$ and A_i is arbitrary, and a decision rule $g = (Q, T)$, such that, for each party $i \in \{1, 2\}$ and every $p \in P = P_2 \times P_1$ the monetary transfer can be written as

$$T_i(p) = F_i(p_i) + p_i Q(p),$$

where $F_i : P_i \longrightarrow \mathbb{R}$ determines the fixed fee.

The difference to price mechanisms is that the message spaces may include other messages than prices. The idea is that, by allowing for arbitrary messages that are never communicated in equilibrium, we can get rid of unwarranted ex post equilibria. We have the following result:

Proposition 4. *The allocation $a = (q, t)$ is strongly ex post implementable if and only if there exists an augmented price mechanism $(P \cup A, g)$ with a unique ex post equilibrium*

$$s^* = (s_1^*, s_2^*) = (p_2, p_1),$$

strongly ex post implementing the allocation.

Proof. Suppose the allocation $a = (q, t)$ is strongly ex post implementable. Then, by definition, there exists a mechanism $\left(\bar{M}, \bar{g}\right)$ such that, for every ex post equilibrium $\bar{s}^*$ in this mechanism we have $\bar{g} \circ \bar{s}^* = a$. Since a is strongly ex post implementable, it is weakly ex post implementable and thus satisfies the two-part tariff structure. Furthermore, by Lemma 3, there exists an augmented price mechanism $(P \cup A, g)$ such that $P_j = p_j(X_i)$ for each $i \in \{1, 2\}$ and $g \circ p = a$, where $p = (p_2, p_1)$. Hence, for any ex post equilibrium $\bar{s}^*$ in $\left(\bar{M}, \bar{g}\right)$ we have that for any $x \in X$:

$$\bar{g}\left(\bar{s}_1^*(x_1), \bar{s}_2^*(x_2)\right) = a(x_1, x_2) = g\left(p_2(x_1), p_1(x_2)\right).$$

But then, there exists an ex post equilibrium $\bar{s}^{**}$ in $\left(\bar{M}, \bar{g}\right)$ with the property that for every $x, x' \in X$:

$$\begin{aligned} p_2(x_1') = p_2(x_1) &\Longrightarrow \bar{s}_1^{**}(x_1') = \bar{s}_1^{**}(x_1), \\ p_1(x_2') = p_1(x_2) &\Longrightarrow \bar{s}_2^{**}(x_2') = \bar{s}_2^{**}(x_2). \end{aligned}$$

The next steps borrow from the proof Theorem 3.2 in Mookherjee and Reichelstein (1990). For each $i \in \{1, 2\}$ define A_i as the set of $\bar{m}_i \in \bar{M}_i$ that do not belong to the range of $\bar{s}_i^{**}$. Furthermore, define a function $\psi_i : P_j \cup A_i \longrightarrow \bar{M}_i$ by setting

$$\psi_i(m_i) = \begin{cases} \bar{s}_i^{**}(x_i) & \text{if } m_i = p_j(x_i) \text{ for } x_i \in X_i, \\ m_i & \text{if } m_i \in A_i. \end{cases}$$

Let $g = \bar{g} \circ \psi$, where $\psi = (\psi_1, \psi_2)$. Clearly p is an ex post equilibrium in $(P \cup A, g)$. To show uniqueness, suppose that $s^* = (s_1^*, s_2^*)$ constitutes an ex post equilibrium in the augmented price mechanism $(P \cup A, g)$. Consider the strategies

$$\bar{s}_i^* (x_i) = \psi_i (s_i^* (x_i))$$

for each $x_i \in X_i$. Since the outcome of s^* in $(P \cup A, g)$ coincides with that of $\bar{s}^*$ in $\left(\bar{M}, \bar{g}\right)$, i.e.

$$\bar{g} \circ \bar{s}^* = \bar{g} \circ \psi \circ s^* = g \circ s^*,$$

it is enough to show that $\bar{s}^*$ is an equilibrium in $\left(\bar{M}, \bar{g}\right)$, because by hypothesis, this original mechanism strongly ex post implements the allocation. Since s^* is an ex post equilibrium in $(P \cup A, g)$ we have for any $i \in \{1, 2\}$, $x \in X$ and $m_i \in P_j \cup A_i$:

$$\begin{aligned} U_i \left(\bar{g}\left(\bar{s}_i^* (x_i), \bar{s}_j^* (x_j)\right); x\right) &= U_i \left(g\left(s_i^* (x_i), s_j^* (x_j)\right); x\right) \text{ by } \bar{g} \circ \bar{s}^* = g \circ s^* \\ &\geq U_i \left(g\left(m_i, s_j^* (x_j)\right); x\right) \text{ by definition of an ex post equilibrium} \\ &= U_i \left(\bar{g}\left(\psi_i (m_i), \psi_j \left(s_j^* (x_j)\right)\right); x\right) \text{ by } g = \bar{g} \circ \psi \\ &= U_i \left(\bar{g}\left(\psi_i (m_i), \bar{s}_j^* (x_j)\right); x\right) \text{ by } \bar{s}_j^* (x_j) = \psi_j \left(s_j^* (x_j)\right). \end{aligned}$$

The result now follows from the fact that the map ψ_i is onto its range $\bar{M}_i$. □

5 Optimal price mechanisms

This section applies the results on implementation to characterize optimal price mechanisms, including the efficient one and the one that maximizes the expected revenue from intermediation, subject to a prior over the signal space. To obtain clear results, we will consider the environment with one-dimensional signal spaces and separable values, making the following assumption:

Assumption 4. *For each* $i \in \{1, 2\}$ *and every* $x \in X$*:*

$$v_i(x_i, x_j) = x_i - e_i x_j,$$

where $e_i \in (0, 1)$*.*

This assumption combines the previous assumptions on negative interdependency and separability, implying that the results on both weak and strong ex post implementation apply. The parameter e_i measures the strength of the negative informational externality. Applying Lemma 1 to this framework, for any weakly ex post implementable allocation and any $x \in X$ we have:

$$q(x_i, x_j) = \begin{cases} 1 & \text{if } x_i > p_i(x_j) + e_i x_j, \\ 0 & \text{if } x_i < p_i(x_j) + e_i x_j. \end{cases}$$

Efficiency The efficient allocation $\overline{a}$ will complete the transaction whenever the total transaction value is positive. It is then straigthforward to calculate that the efficient transaction price functions are given by

$$\overline{p}_i(x_j) = -\left(e_j + \frac{1 - e_i}{1 - e_j}\right) x_j.$$

Note that the term in the brackets is positive, implying that the price functions are strictly decreasing, as they should be. We are now ready to characterize efficient price mechanisms:

Proposition 5. *The efficient price mechanism* $(P, \overline{g})$ *has message spaces* $P_i = \overline{p}_i(X_j)$ *and a decision rule* $\overline{g}$*, such that, for each* $i \in \{1, 2\}$ *and every* $p \in P$*:*

$$\overline{T}_i(p) = F_i(p_i) + p_i \overline{Q}(p),$$

where $F_i : P_i \longrightarrow \mathbb{R}$ *is arbitrary and*

$$\overline{Q}(p) = \begin{cases} 1 & \textit{if } \left(1 - e_i + e_j - e_j^2\right) p_j + \left(1 - e_j + e_i - e_i^2\right) p_i < 0, \\ 0 & \textit{if } \left(1 - e_i + e_j - e_j^2\right) p_j + \left(1 - e_j + e_i - e_i^2\right) p_i > 0. \end{cases}$$

Proof. By inverting $p_i = \overline{p}_i(x_j)$ we obtain

$$x_j = \overline{p}_i^{-1}(p_i) = -\frac{(1 - e_j) p_i}{e_j(1 - e_j) + 1 - e_i},$$

so that, by using $\overline{Q} = q \circ \left(\overline{p}_2^{-1}, \overline{p}_1^{-1}\right)$,

$$\overline{q}(x_i, x_j) = \begin{cases} 1 & \text{if } (1 - e_j) x_i + (1 - e_i) x_j > 0, \\ 0 & \text{if } (1 - e_j) x_i + (1 - e_i) x_j < 0, \end{cases}$$

can be equivalently written as

$$\overline{Q}(p) = \begin{cases} 1 & \text{if } \left(1 - e_i + e_j - e_j^2\right) p_j + \left(1 - e_j + e_i - e_i^2\right) p_i < 0, \\ 0 & \text{if } \left(1 - e_i + e_j - e_j^2\right) p_j + \left(1 - e_j + e_i - e_i^2\right) p_i > 0. \end{cases}$$

□

Under asymmetric informational externalities, the intermediary puts more weight on the price chosen by the party whose informational exterality is smaller. In the symmetric case $e_i = e$ we have

$$\overline{Q}(p) = \begin{cases} 1 & \text{if } p_j + p_i < 0, \\ 0 & \text{if } p_j + p_i > 0, \end{cases}$$

implying that, to achieve efficiency, the intermediary must execute the transaction if and only if the total price is negative.

Maximizing expected revenue To characterize the revenue-maximizing price mechanism, we suppose that each signal x_i is independently distributed according to a cdf H_i with a positive and continuous density function h_i. Furthermore, we impose the following regularity assumption:

Assumption 5. *For each $i \in \{1,2\}$ the externality-adjusted virtual type*

$$(1-e_j)\,x_i - \frac{1-H_i\left(x_i\right)}{h_i\left(x_i\right)}$$

is strictly increasing.

Let us first characterize the revenue-maximizing allocation, $\hat{a}$, abstracting away from the fixed fees, as these are determined by appropriate individual rationality constraints. It is convenient to express the expected revenue from intermediation as the total expected surplus less the information rent left to the parties. Applying Lemma 1, the expected utility of party i conditional on x_i writes

$$\begin{aligned} E_{x_j}\left[U_i\left(a\left(x\right);x\right)\right] &= \int_{\underline{x}_j}^{\overline{x}_j} \left[v_i\left(x_i,x_j\right) - p_i\left(x_j\right)\right] q\left(x_i,x_j\right) h_j\left(x_j\right) \mathrm{d}x_j \\ &= \int_{p_j(x_i)+e_j x_i}^{\overline{x}_j} \left[x_i - e_i x_j - p_i\left(x_j\right)\right] h_j\left(x_j\right) \mathrm{d}x_j . \end{aligned}$$

Taking the expectation over x_i we obtain the unconditional expected utility:

$$\begin{aligned} E_{x_i}\left\{E_{x_j}\left[U_i\left(a\left(x\right);x\right)\right]\right\} &= \int_{\underline{x}_i}^{\overline{x}_i} \int_{p_j(x_i)+e_j x_i}^{\overline{x}_j} \left[x_i - e_i x_j - p_i\left(x_j\right)\right] h_j\left(x_j\right) h_i\left(x_i\right) \mathrm{d}x_j \mathrm{d}x_i \\ &= \int_{\underline{x}_j}^{\overline{x}_j} \int_{p_i(x_j)+e_i x_j}^{\overline{x}_i} \left[x_i - e_i x_j - p_i\left(x_j\right)\right] h_i\left(x_i\right) h_j\left(x_j\right) \mathrm{d}x_i \mathrm{d}x_j , \end{aligned}$$

where the second equality follows from Lemma 1 and Fubini's theorem, allowing us to switch the order of integration. Using integration by parts in the brackets then yields

$$E_{x_i}\left\{E_{x_j}\left[U_i\left(a\left(x\right);x\right)\right]\right\} = \int_{\underline{x}_j}^{\overline{x}_j}\int_{p_i(x_j)+e_ix_j}^{\overline{x}_i}\left[1-H_i\left(x_i\right)\right]h_j\left(x_j\right)\mathrm{d}x_i\mathrm{d}x_j.$$

By symmetry, the unconditional expected utility of party j writes:

$$\begin{aligned}E_{x_j}\left\{E_{x_i}\left[U_j\left(a\left(x\right);x\right)\right]\right\} &= \int_{\underline{x}_i}^{\overline{x}_i}\int_{p_j(x_i)+e_jx_i}^{\overline{x}_j}\left[1-H_j\left(x_j\right)\right]h_i\left(x_i\right)\mathrm{d}x_j\mathrm{d}x_i \\ &= \int_{\underline{x}_j}^{\overline{x}_j}\int_{p_i(x_j)+e_ix_j}^{\overline{x}_i}\left[1-H_j\left(x_j\right)\right]h_i\left(x_i\right)\mathrm{d}x_i\mathrm{d}x_j,\end{aligned}$$

switching the order of integration again. Summing the unconditional expected utilities, we get

$$\int_{\underline{x}_j}^{\overline{x}_j}\int_{p_i(x_j)+e_ix_j}^{\overline{x}_i}\left[\frac{1-H_i\left(x_i\right)}{h_i\left(x_i\right)}+\frac{1-H_j\left(x_j\right)}{h_j\left(x_j\right)}\right]h_i\left(x_i\right)h_j\left(x_j\right)\mathrm{d}x_i\mathrm{d}x_j.$$

Subtracting this information rent from the total expected surplus obtains the expected revenue from intermediation:

$$\int_{\underline{x}_j}^{\overline{x}_j}\int_{p_i(x_j)+e_ix_j}^{\overline{x}_i}\sum\left[\left(1-e_j\right)x_i-\frac{1-H_i\left(x_i\right)}{h_i\left(x_i\right)}\right]h_i\left(x_i\right)h_j\left(x_j\right)\mathrm{d}x_i\mathrm{d}x_j.$$

The revenue-maximizing price schedule $\hat{p}_i\left(\cdot\right)$ maximizes the integrand pointwise. Taking the first-order condition we obtain:

$$\left(1-e_j\right)\left(\hat{p}_i\left(x_j\right)+e_ix_j\right)-\frac{1-H_i\left(\hat{p}_i\left(x_j\right)+e_ix_j\right)}{h_i\left(\hat{p}_i\left(x_j\right)+e_ix_j\right)} = -\left(1-e_i\right)x_j+\frac{1-H_j\left(x_j\right)}{h_j\left(x_j\right)}.$$

Note that the price function is strictly decreasing by Assumption 5. Furthermore, observe that by inverting the first-order condition we obtain the optimal price $\hat{p}_j\left(x_i\right)$ for party j. We are now ready to cast the following result:

Proposition 6. *The revenue-maximizing price mechanism* $\left(P,\hat{g}\right)$ *has message spaces*

$P_i = \hat{p}_i(X_j)$ and a decision rule $\hat{g}$, such that, for each $i \in \{1,2\}$ and every $p \in P$:

$$\hat{T}_i(p) = F_i(p_i) + p_i \hat{Q}(p),$$

where $F_i : P_i \longrightarrow \mathbb{R}$ is arbitrary and

$$\hat{Q}(p) = \begin{cases} 1 & \textit{if } \hat{p}_j^{-1}(p_j) > p_i + e_i \hat{p}_i^{-1}(p_j), \\ 0 & \textit{if } \hat{p}_j^{-1}(p_j) < p_i + e_i \hat{p}_i^{-1}(p_j). \end{cases}$$

Proof. By inverting $p_j = \hat{p}_j(x_i)$ we have $x_i = \hat{p}_j^{-1}(p_j)$ for each $i \in \{1,2\}$ and therefore, by using $\hat{Q} = q \circ \left(\hat{p}_2^{-1}, \hat{p}_1^{-1}\right)$,

$$q(x_i, x_j) = \begin{cases} 1 & \text{if } x_i > \hat{p}_i(x_j) + e_i x_j, \\ 0 & \text{if } x_i < \hat{p}_i(x_j) + e_i x_j, \end{cases}$$

can be equivalently written as

$$\hat{Q}(p) = \begin{cases} 1 & \text{if } \hat{p}_j^{-1}(p_j) > p_i + e_i \hat{p}_i^{-1}(p_j), \\ 0 & \text{if } \hat{p}_j^{-1}(p_j) < p_i + e_i \hat{p}_i^{-1}(p_j). \end{cases}$$

□

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