Competing for Audience while Contracting on Advertising Sales*

Matias Pietola†

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Abstract

The paper studies advertising sales' cooperation between media platforms (television or radio channels, newspapers, etc.) that compete over content offered to consumers. A sales representation agreement, whereby one of the platforms delegates its advertising sales to another platform, in exchange for a fee per subscriber, not only increases the price of advertising, but also reduces content investment. Revenue sharing leads to even less content investment, as the platforms free-ride on the content paid by the other.

Keywords: media platforms, advertising, content provision, cooperation.

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†The Finnish Competition and Consumer Authority; Toulouse School of Economics (TSE), University of Toulouse 1 Capitole, Toulouse, FRANCE, matias.pietola@iki.fi.

1 Introduction

With Google and Facebook capturing around 60 percent of the global digital advertising spending (eMarketer, March 2018), the more traditional content providers are struggling to retain their share of the market (The Economist, Feb 16th 2019). Those relying purely on advertising revenue to cover the cost of their content face a particularly difficult situation, as tougher competition on the advertising side reduces their ad revenue and hence the profitability of content investments. Yet cutting down on these investments may trigger a vicious circle where poorer content attracts fewer consumers, and the resulting smaller audience is less attractive for the advertisers making it more difficult for them to compete for their money.

One potential solution to this problem is to cooperate on the advertising side. Contractual arrangements, such as advertising sales representation agreements, whereby one media platform acts as a representative in selling ads to the advertisers, are common.[1] Cooperation may also take the form of a joint ad sales house, through which advertising revenue between different media platforms can be shared.[2] These arrangements rarely extend to the other side of the market, where the platforms compete for the consumers of content. Thus, content choices remain independent, which prompts the question of how cooperation on the advertising side affects platform competition on the content and consumer side.

To address this question, we develop a setting in which two initially symmetric media platforms provide free content to consumers and sell ads. The platforms invest to improve their content in order to attract consumers, who are heterogeneous in their preferences. The advertising revenue is proportional to market coverage. Platform com-

[1]For instance, MTG manages advertising sales for Disney in Scandinavia and Sky Media acts as a sales agent for its own channels and several rival ones in Germany and in the UK. Similar arrangements can be found in other EU countries, including the Netherlands and Spain.

[2]BrandDeli is a joint sales house between Discovery, Viacom, and FOX in the Netherlands. NextRégie is a French sales house marketing various TV channels.

petition affects investment in content in two ways. First, competition on the advertising side reduces the per customer advertising revenue, which limits investment incentives. Conversely, competition on the consumer side involves business stealing, as the expansion in one platform's customer base is partly made at the expense of the rival. This negative externality tends to induce platforms to invest more than the amount that maximizes industry profit.

To alleviate the first effect, the platforms can coordinate on their advertising sales. Coordination may take various forms, including representation agreements and revenue sharing arrangements. We first consider a situation where the platforms enter into a representation agreement based on a two-part tariff and consumers single-home (that is, they subscribe to, at most, one platform). We then extend the analysis to multi-homing consumers and more sophisticated contractual arrangements.

In a representation agreement the sales agent markets the ads of both platforms and pays the principal a wholesale price for each of its customers. Furthermore, the parties can share revenue though a fixed fee. When consumers single-home, the agreement has no impact on the per consumer advertising revenue, as each platform controls access to its own customers. The agreement, however, softens competition on the consumer side, which harms consumers.

The intuition is straightforward. By agreeing on a wholesale price that is lower than the per consumer advertising revenue, the platforms will reduce their incentives to invest in content. The principal invests less, simply because its marginal advertising revenue is now determined by the wholesale price. The sales agent invests less, because it earns a margin on the principal's consumers, which makes stealing them less profitable. The agreement thus softens competition for the audiences.

The agreement allows the platforms to increase their joint profit, but does not enable them to maximize it: the optimal agreement instead induces the sales agent to over-invest and the principal to under-invest, compared to the industry-wide monopoly outcome. This is because the agreement provides a single instrument, namely the wholesale price, to achieve two targets: attracting new consumers and avoiding fierce competition. The optimal wholesale price balances these two goals.

In practice, however, consumers patronize multiple platforms. Allowing for multi-homing by consumers shifts competition from the consumer side to the advertising side of the market: the platforms no longer have monopolies over their audiences, which reduces the ad revenue per consumer, but this, in turn, softens competition for market share.

The representation agreement eliminates competition for the advertisers and increases advertising revenue. We show, however, that this does not lead to more investment in content. In the limit case of independent demands for the two platforms, the level of content is the same in the competitive and (industry-wide) monopoly benchmarks. The optimal agreement then simply eliminates competition for the advertising, but has no effect on content. By contrast, whenever the platforms offer (imperfect) substitutes, the monopolistic investments are lower than in the competitive benchmark, and the optimal agreement still induces the principal to invest too little and the sales agent to invest too much.

More sophisticated agreements do enable the firms to achieve the monopoly outcome. The first possibility is to augment the two-part tariff by another wholesale price, which the principal obtains for each single-homer on the sales agent's platform. Charging a smaller wholesale price for these single-homers enables the firms to invest as an industry-wide monopoly. By contrast, merely sharing the advertising revenue (e.g., by setting a joint ad sales house) fails to achieve the monopoly outcome. This increases the price of advertising by the same amount as the representation agreement, but leads to even worse content levels, as each platform free-rides on the content paid by the other.

Related literature. This paper relates to the literature on media platforms, initiated by Anderson and Coate (2005). They show that a merger reduces competition for ad-averse consumers, which increases the amount of advertising and lowers its price. This result flips around if consumers multi-home or there is congestion in advertising (Anderson et al., 2012; Ambrus et al., 2016; Anderson et al., 2019). If consumers multi-home, the platforms no longer have a monopoly over their own audiences: a merger then eliminates competition on the advertising side. If, instead, consumers register only a fraction of the total number of ads they see, more ads on one platform creates a congestion externality on the other platform, which is internalized by a merger. In both cases a merger reduces the volume of advertising and increases its price.

In a related manner, Anderson and Peitz (2015) show that if content is offered free of charge, a merger increases the amount of advertising, which harms consumers but the effect on advertisers is ambiguous. However, if there are too many platforms, competition between them may reduce the quality of content and also harm consumers (Liu et al., 2004). Peitz and Valletti (2008) show that free-to-air television involves more advertising than paid television if viewers strongly dislike advertising. They also show that free-to-air television broadcasters offer less differentiated content.

Using a structural model to simulate the effects of a blocked newspaper merger in Minneapolis, Fan (2013) shows that the merger would have increased subscription fees and reduced circulation, harming both readers and advertisers. Chandra and Collard-Wexler (2009) show instead that a merger between two Canadian newspapers did not increase prices on either side of the market. Sweeting (2010) studies the effects of a merger in the music radio industry on product positioning and shows that, post merger, the stations re-position themselves to avoid cannibalizing their audiences, which led them to gain market share at the expense of their rivals.

Instead of studying the effects of a full merger, this paper studies the implications of cooperation on advertising and, more specifically, its impact on content provision. It shows that advertising sales representation agreements – where the sales agents pays the principal a wholesale price for each consumer the principal attracts – fail to implement the same outcome as a merger. Nevertheless, this harms advertisers, who face higher prices, and consumers; particularly for those of the principal, who obtain even worse content than in case of a full merger. Interestingly, these insights carry over, regardless of whether consumers single- or multi-home.

These results contrast with those obtained by Dewenter et al. (2011) on semi-collusion in newspaper markets. According to their result, collusion on the advertising side increases the price of advertising, which in turn intensifies competition for the readers, as readers bring more advertising revenue. As a result, colluding on advertising benefits readers through lower subscription fees.

The paper is organized as follows. The next section presents the setting and characterizes the monopoly and competitive benchmarks. Section 3 studies cooperation based on a wholesale price when consumers single-home. Section 4 extends the analysis by allowing consumers to multi-home. Section 5 considers alternative forms of cooperation. All proofs are relegated to the Appendix.

2 Baseline model

2.1 Setting

We consider a two-sided media market, in which two symmetric platforms, A and B, compete for consumers by offering them free content and obtain revenue from advertising. By improving its content, a platform attracts more consumers, which increases its advertising capacity at the expense of its rival.

Specifically, content of quality $q \geq 0$ can be obtained at cost $c(q)$, where $c'(q) > 0$, $c''(q) > 0$ and $c(0) = c'(0) = 0$. Better content is thus costlier to improve, whereas zero content costs nothing and can be improved at a small cost.

There is a unit mass of consumers with heterogeneous preferences for content in the spirit of Perloff and Salop (1985). An individual consumer is characterized by a two-dimensional type $\theta = (\theta_A, \theta_B)$ that is uniformly distributed on the unit square. For the time being, we assume that consumers single-home.[3] By consuming q_i from platform i, a type θ consumer obtains utility $q_i - \theta_i$. Each consumer has an outside option, normalized to zero, and a neutral attitude towards advertising.

Advertisers are willing to pay a fixed amount $\rho > 0$ per unique ad impression and nothing for additional impressions. A possible micro-foundation, following Anderson and Coate (2005), is that advertising their goods enables producers to reach consumers and generate a match worth of ρ. Thus, a producer earns ρN_i by advertising to N_i consumers on platform i and ρN by advertising on both platforms, where $N = N_A + N_B$ denotes the total audience. Without advertising the producer earns zero profit.

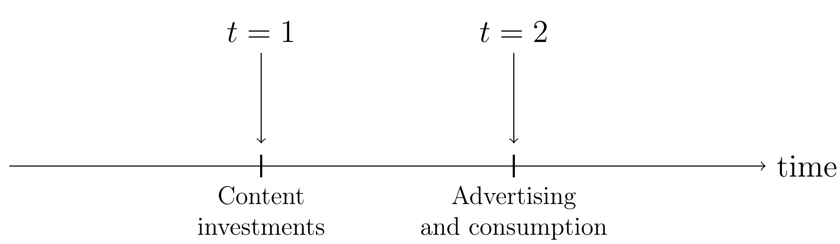

Figure 1: Timing of the game.

Figure 1 presents the timing. The platforms first invest in content simultaneously and independently. Investment costs are sunk and the resulting contents are publicly observable. The platforms then simultaneously choose the prices for their ads, producers decide where to advertise their goods, and consumers decide which platform to

[3] We first focus on the case where consumers use a single platform, and consider the case of multi-homers later on.

join, if any. For the sake of exposition, we assume that consumption and advertising decisions are taken simultaneously, but the sequence of those decisions does not affect the analysis.[4]

Each platform seeks to maximize its own profit, equal to the revenue net of the cost of content. The equilibrium concept is subgame perfect Nash equilibrium. The following assumption ensures equilibrium existence, uniqueness and stability:

Assumption 1. *$c''(q) > 2\rho$ for any q.*

2.2 Consumer choice and advertising

Consumers base their decisions on content qualities: a consumer of type θ chooses platform i iff $\theta_i \leq \min\{\theta_j - q_j, 0\} + q_i$. Integrating over consumer types, the number of consumers on platform i is

$$N_i(q_A, q_B) = \int_{q_j - \min\{q_A, q_B\}}^{\min\{q_j, 1\}} (\theta_j - q_j + q_i)\,\mathrm{d}\theta_j + (1 - \min\{q_j, 1\}) \min\{q_i, 1\}. \quad (1)$$

The first term captures the consumers who are interested in the rival and the second one captures those who are not. As long as no content exceeds one, (1) simplifies to

$$N_i(q_A, q_B) = \min\{q_A, q_B\} q_i - \frac{\min\{q_A^2, q_B^2\}}{2} + (1 - q_j) q_i$$

and the total audience is

$$N(q_A, q_B) = q_A + q_B - q_A q_B.$$

Figure 2 illustrates this consumer demand.

[4] Indeed, consumer responses are uniquely pinned down by the contents. It does not matter if the choices are observed or not; what matters is full information about the contents and prices.

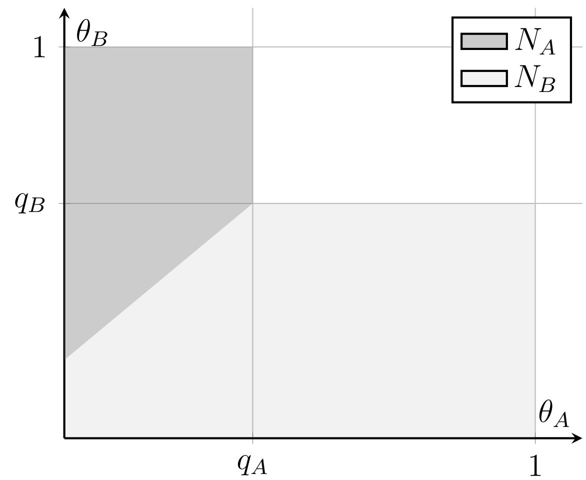

Figure 2: Single-homing with $q_B = \frac{3}{5} > q_A = \frac{2}{5}$.

Remark 1. The consumer demands are continuously differentiable:

$$\frac{\partial N_i}{\partial q_i}(q_A, q_B) = 1 - q_j + \min\{q_A, q_B\},$$
$$\frac{\partial N_i}{\partial q_j}(q_A, q_B) = -\min\{q_A, q_B\}.$$

In particular, we have

$$\frac{\partial N}{\partial q_i}(q_A, q_B) = 1 - q_j.$$

Observing the contents and anticipating consumer demand, producers choose where to advertise their products. Due to single-homing, the platforms have a monopoly over their own audiences and a producer is willing to pay $\rho N_i(q_A, q_B)$ to advertise on platform i independently of its purchase decision on platform j. Thus, in this baseline model, the platforms price at ρ and producers purchase ad space from both platforms, earning zero profit.

2.3 Monopoly

An industry-wide monopoly, owning both platforms, chooses the contents in order to maximize the total advertising revenue net of the total cost of content:

$$\Pi(q_A, q_B) = \rho N(q_A, q_B) - c(q_A) - c(q_B).$$

By Assumption 1, the industry profit is strictly concave.[5] The optimization problem thus boils down to equalizing the marginal cost of content with the overall marginal return on investment; the monopoly contents, (q_A^m, q_B^m), are thus given by

$$c'(q_i^m) = \rho \frac{\partial N}{\partial q_i}(q_A^m, q_B^m) = \rho\left(1 - q_j^m\right).$$

The symmetry of the problem implies that the monopolistic contents, being unique, are also symmetric: $q_A^m = q_B^m = q^m$. It can be checked that $q^m \in (0,1)$ is strictly increasing with ρ: higher ad revenue per consumer induces the monopoly to invest more in content, to attract more consumers. However, the marginal return is less than this, because the monopoly internalizes cannibalization between the contents and cares about expanding the total audience.

2.4 Competition

Competing platforms do not internalize this externality. Each platform i seeks to maximize its own advertising revenue net of its cost of content:

$$\pi_i(q_A, q_B) = \rho N_i(q_A, q_B) - c(q_i).$$

[5]We have $\frac{\partial^2 \Pi}{\partial q_i^2} = -c''(q_i) < 0$ and the Hessian is equal to $c''(q_A)\, c''(q_B) - \rho^2 > 0$.

By Assumption 1 the profit is strictly concave, so the first-order condition is necessary and sufficient for optimality. The best response to rival's content, $R_i(q_j)$, is thus characterized by

$$c'(R_i(q_j)) = \rho(1 - q_j) + \rho \min\{q_j, R_i(q_j)\}.$$

This can be written as:

$$c'(R_i(q_j)) = \begin{cases} \rho & \text{if } c'(q_j) \leq \rho, \\ \rho[1 - q_j + R_i(q_j)] & \text{if } c'(q_j) > \rho. \end{cases}$$

In the equilibrium, which is illustrated in Figure 3 and constitutes a special case of Lemma 1 below, the platforms choose q^c to equalize the marginal cost of content with the ad revenue per consumer, neglecting the negative impact on the rival: $c'(q^c) = \rho$. This negative externality leads to better content than the platforms would jointly prefer: $q^c > q^m$. Better content is obviously good for the consumers.

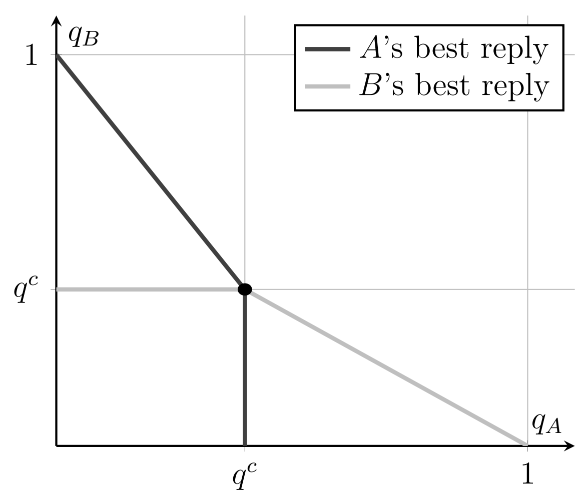

Figure 3: Competitive equilibrium.

3 Cooperation

3.1 Extended setting

We now allow the platforms to cooperate on selling their ads. Specifically, cooperation takes the form of a representation agreement (w, f), according to which A represents both platforms for the sale of advertising: A then chooses the monopoly price ρN_i for each platform i and obtains the total advertising revenue. In return, A pays B a two-part tariff $f + wN_B$ (with possibly negative prices). That is, B gets a fixed fee f and a wholesale price w from each consumer it attracts. If there is no agreement, the platforms price their advertising slots independently.

In what follows, we will remain agnostic about the exact nature of the negotiation, and simply assume that, as the platforms can share their profits through a fixed fee, they seek to maximize their joint profit. We are interested in the implications of the resulting agreement on content provision.

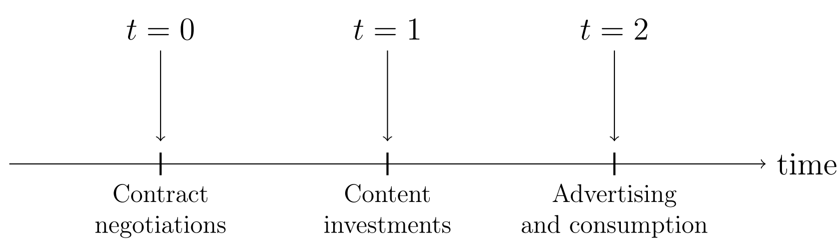

Figure 4: Timing of the extended game.

Figure 4 presents the timing of this extended setting.

3.2 Content investments

The aim of this section is to characterize the continuation equilibrium following any given representation agreement (w, f). The competitive equilibrium corresponds to the "null" agreement $(\rho, 0)$. Any agreement with $w < 0$ is equivalent to $w = 0$ as B then chooses zero content to avoid losses. Furthermore, agreeing on $w > \rho$ is never optimal, because both platforms would invest more than in the competitive equilibrium, taking them even farther away from the monopoly outcome.

We can thus restrict attention to representation agreements in which $w \in [0, \rho]$. Given any such agreement, A maximizes the total advertising revenue net of its own content cost and the payment to B, whereas B seeks to maximize the payment less its content cost:

$$\pi_A (q_A, q_B; w, f) = \rho N (q_A, q_B) - w N_B (q_A, q_B) - f - c (q_A) ,$$
$$\pi_B (q_A, q_B; w, f) = w N_B (q_A, q_B) + f - c (q_B) .$$

Using the expressions in Remark 1 above, we have:

$$\frac{\partial N_B}{\partial q_A} (q_A, q_B) = - \min \{q_A, q_B\} ,$$
$$\frac{\partial N_B}{\partial q_B} (q_A, q_B) = (1 - q_A) + \min \{q_A, q_B\} .$$

The game has a unique and stable equilibrium:

Lemma 1. *For any agreement (w, f) with $w \in [0, \rho]$ there exists a unique and stable continuation equilibrium $(q_A (w), q_B (w))$ such that:*

$$c' (q_A (w)) = \rho - (\rho - w) q_B (w) ,$$
$$c' (q_B (w)) = w - w [q_A (w) - q_B (w)] ,$$

with $0 < q_B (w) < q_A (w) < q^c$ for any $w \in (0, \rho)$. In particular, $q_B (0) = 0$ and $q_A (0) = q_A (\rho) = q_B (\rho) = q^c$.

The representation agreement affects the content investment incentives of both platforms by changing their marginal returns to investment. Instead of the full ad revenue per consumer, B gets the wholesale price for each consumer it attracts. The difference is pocketed by A who also earns a margin on B's customers. Nonetheless, unless the wholesale price is zero, A still gets a higher ad revenue from its own consumers, implying that cooperation does not eliminate all competition. How much competition is left depends on the wholesale price. We have the following comparative statics:

Lemma 2. *For any $w \in (0, \rho)$ the equilibrium satisfies*

$$q_B'(w) > 0 \text{ and } q_A'(w) + q_B'(w) > 0.$$

Furthermore, $q_A'(0) < 0$ and $q_A'(\rho) > 0$.

Figure 5 illustrates how the cooperative equilibrium departs from the competitive one by reducing the contents of both platforms. By setting the wholesale price below the ad revenue per consumer, the platforms directly reduce B's incentive to improve its content, because its marginal return to investment is smaller. This relationship is monotonic: a decrease in the wholesale price induces a decrease in the content chosen by B.

Furthermore, as A makes money on B's audience as well, content investments that steal part of it are less profitable for A. However, a decrease in the wholesale price does not always reduce the content chosen by A in equilibrium. Decreasing an already low wholesale price will actually cause A to invest more, because content choices are strategic substitutes. When the wholesale price equals zero, there are no consumers to steal, as B chooses zero content and has no audience. In this special case A selects the same content as it would under platform competition.

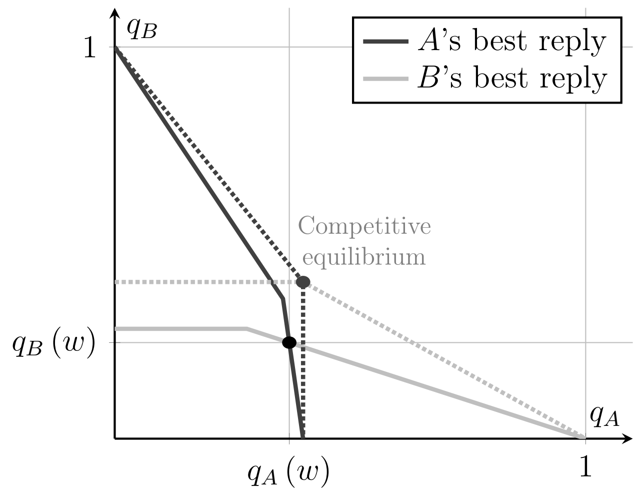

Figure 5: Cooperative equilibrium for $w \in (0, \rho)$.

3.3 Optimal contract

As mentioned, the platforms choose the wholesale price so as to maximize the resulting joint profit $\Pi(w)$. By the envelope theorem:

$$\begin{aligned}\Pi'(w) &= \frac{\partial \pi_A}{\partial q_B} q_B'(w) + \frac{\partial \pi_B}{\partial q_A} q_A'(w) + \overbrace{\frac{\partial \pi_A}{\partial w} + \frac{\partial \pi_B}{\partial w}}^{=0} \\ &= (\rho - w)\left[1 - q_A(w)\right] q_B'(w) - w q_B(w)\left[q_A'(w) + q_B'(w)\right].\end{aligned}$$

From Lemma 2 we then have $\Pi'(0) > 0 > \Pi'(\rho)$. It follows that the optimal wholesale price satisfies $w^* \in (0, \rho)$ and is characterized by the first-order condition $\Pi'(w^*) = 0$. Building on this yields:

Proposition 1. *When consumers single-home, the optimal contract (w^*, f) keeps the monopoly price of advertising, and yields asymmetric contents,*

$$q_B(w^*) < q^m < q_A(w^*) < q^c,$$

harming the consumers of content on both platforms. The principal's content is worse *than in the monopoly benchmark, whereas the sales agent's content is better but still worse than the competitive one.*

It is instructive to look at the marginal advertising revenues when both platforms choose the same content q. By slightly increasing its content, A increases its revenue by $\rho(1-q)+wq$, where $1-q$ represents the number of new consumers and q the number of consumers switching from B, which reduces the payment. When, instead, B improves its content, its marginal revenue is $w(1-q)+wq=w$, where $1-q$ corresponds again to the new consumers and q to the switchers from A, but both effects now increase the payment from A.

A decrease in the wholesale price thus reduces A's incentives to steal consumers from B and B's incentives to steal consumers from A, but in addition it also reduces B's incentives to expand the total audience; by contrast, the wholesale price has no impact on A's incentives to expand this total audience. Due to this asymmetry, the platforms fail to implement the industry-wide monopoly outcome.

Indeed, A has the incentive to choose the monopoly content only if the platforms agree on a zero wholesale price. However, this totally removes B's incentive to improve its content and expand the total audience. Thus, there is a trade-off. On the one hand, the platforms would like to keep the wholesale price high enough to encourage sufficient content investment from B, so as to attract new consumers. On the other hand, they would like to keep the wholesale price low to relax competition between them. The wholesale price offers one tool to pursue these two objectives.

The optimal wholesale price balances the trade-off between softening mutual competition and expanding the audience. As Proposition 1 shows, this induces an asymmetric equilibrium in which A invests too much and B too little compared to the monopoly outcome. However, both invest less than in the competitive equilibrium. Hence, the agreement harms consumers.

4 Multi-homing consumers

4.1 Shared audiences

When all consumers single-home, there is no competition on the advertising side of the market, because the platforms control the access to their consumers' attention. This implies that cooperation on the advertising side has no impact on the price of advertising, allowing us to look at the effects of this cooperation purely on content investments: by relaxing competition for the audience, the cooperation harms consumers due to poorer content. We now extend the baseline model to allow for multi-homing on the consumer side, and thus competition on the advertising side.

A simple modeling approach is to assume that, by consuming content from both platforms, a consumer obtains the sum of the two surpluses, $q_A + q_B - \theta_A - \theta_B$. A consumer then chooses platform i iff $q_i \geq \theta_i$, independently of q_j and θ_j. It follows that, by improving its content, a platform does not reduce the audience of the rival platform, but simply increases the number of multi-homers. We refer to this as the case of "independent values".

More generally, though, we allow that the content offered by the two platforms can be partially substitutable. To capture this, we will assume that consumers discount their less preferred content by some $\sigma \in [0, 1]$. A consumer with a preference for q_i then multi-homes iff $\sigma q_j \geq \theta_j$. The case of $\sigma = 1$ corresponds to independent values, whereas $\sigma = 0$ matches the baseline model with no multi-homing. One interpretation is that the contents offered by the platforms are partly redundant in the eyes of the consumers, and that the degree of redundancy is decreasing in σ.

In what follows we will decompose each audience N_i into multi-homers, $\hat{N}_{AB}$, and single-homers, $\hat{N}_i = N_i - \hat{N}_{AB}$. As in the baseline model, advertisers are willing to pay a fixed amount $\rho > 0$ per unique ad impression and nothing for additional impressions.

The incremental value of advertising on platform i is therefore

$$\rho \hat{N}_i = \rho N - \rho N_j.$$

4.2 Consumer choice and incremental pricing of ads

It is useful to first characterize the number of exclusive consumers. A consumer of type θ single-homes on platform i iff $\theta_i \leq \min\{\theta_j - q_j, 0\} + q_i$ and $\sigma q_j < \theta_j$. Integrating over consumer types, platform i's exclusive consumer base is given by:

$$\hat{N}_i\left(q_A, q_B; \sigma\right) = \int_{q_j - \min\{q_i, (1-\sigma) q_j\}}^{\min\{q_j, 1\}} \left(\theta_j - q_j + q_i\right) \mathrm{d}\theta_j + \left(1 - \min\{q_j, 1\}\right) \min\{q_i, 1\}.$$

When $\max\{q_A, q_B\} \leq 1$, this simplifies to

$$\hat{N}_i\left(q_A, q_B; \sigma\right) = \min\{q_i, (1-\sigma) q_j\} q_i - \frac{\min\left\{q_i^2, (1-\sigma)^2 q_j^2\right\}}{2} + \left(1 - q_j\right) q_i.$$

Figure 6 illustrates the impact of σ on consumer demands, for $(q_A, q_B) = \left(\frac{2}{5}, \frac{3}{5}\right)$. B thus offers better quality, which expands its *core* customer base at the expense of A; that is, a larger proportion of consumers favor B if they have to single-home. This implies that B attracts indeed a larger proportion of single-homers, but also that A's core consumers have a relatively strong preference for that platform. Still, for $\sigma > \frac{1}{3}$ the benefit from multi-homing is sufficiently high that some of A's core consumers also consume the content of B. By contrast, when $\sigma < \frac{1}{3}$, none of A's core consumers multi-home.

The number of shared consumers is

$$\hat{N}_{AB}\left(q_A, q_B; \sigma\right) = N\left(q_A, q_B\right) - \hat{N}_A\left(q_A, q_B; \sigma\right) - \hat{N}_B\left(q_B, q_A; \sigma\right).$$

Remark 2. As in the baseline model, the consumer demands are continuously differen-

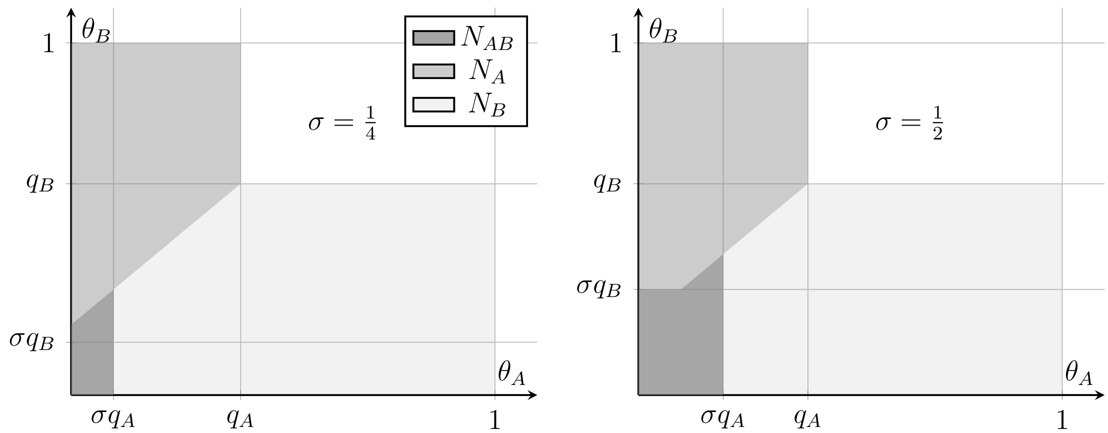

Figure 6: Multi-homing with $q_B = \frac{3}{5} > q_A = \frac{2}{5}$.

tiable. We may thus write

$$\frac{\partial \hat{N}_i}{\partial q_i}(q_A, q_B; \sigma) = 1 - q_j + \min\{q_i, (1-\sigma) q_j\},$$
$$\frac{\partial \hat{N}_i}{\partial q_j}(q_A, q_B; \sigma) = -\sigma q_i - (1-\sigma)\min\{q_i, (1-\sigma) q_j\}.$$

Furthermore:

$$\frac{\partial \hat{N}_{AB}}{\partial q_i}(q_A, q_B; \sigma) = \sigma q_j + (1-\sigma)\min\{q_j, (1-\sigma) q_i\} - \min\{q_i, (1-\sigma) q_j\}.$$

Multi-homing triggers competition on the advertising side of the market. We have the following lemma:

Lemma 3. *(Anderson et al., 2017) Without cooperation, there exists a unique continuation equilibrium, where the platforms price at their incremental values and producers purchase advertising space from both platforms.*

Each platform i gets advertising revenue $\rho \hat{N}_i(q_A, q_B; \sigma)$ and producers make a profit $\rho \hat{N}_{AB}(q_A, q_B; \sigma)$. The total gains from advertising remain the same, and the only difference is that due to competition the producers get a share of it.

We will next revisit the benchmarks of monopoly and competition, before examining cooperation between the platforms.

4.3 Benchmarks revisited

An industry-wide monopoly can bundle the advertising slots of the two platforms to eliminate competition between them. Multi-homing on the consumer side of the market does not affect the monopoly benchmark, because an industry-wide monopoly only cares about expanding the total audience, maximizing the total advertising revenue net of the content costs. Hence, $q_A^m = q_B^m = q^m$ still characterizes the monopoly outcome, regardless of the degree of redundancy in contents.

However, multi-homing does affect the competitive equilibrium, as competing platforms cannot monetize multi-homers and thus invest in content to maximize

$$\pi_i\left(q_A, q_B\right) = \rho \hat{N}_i\left(q_A, q_B; \sigma\right) - c\left(q_i\right).$$

The first-order condition is necessary and sufficient for optimality. The best response is given by

$$c'\left(R_i\left(q_j\right)\right) = \begin{cases} \rho & \text{if } c'\left(\frac{q_j}{1-\sigma}\right) \leq \rho, \\ \rho\left[1 - q_j + \left(1-\sigma\right) R_i\left(q_j\right)\right] & \text{if } c'\left(\frac{q_j}{1-\sigma}\right) > \rho. \end{cases} \tag{2}$$

We have the following lemma:

Lemma 4. *Without cooperation there exists a unique and stable continuation equilibrium, where both platforms choose $q^c\left(\sigma\right)$ characterized by*

$$c'\left(q^c\left(\sigma\right)\right) = \rho\left(1 - \sigma q^c\left(\sigma\right)\right).$$

Furthermore, $(q^c)'(\sigma) < 0$ *and* $q^c(0) = q^c > q^c(1) = q^m$.

Figure 7 illustrates the competitive equilibrium. As shared consumers yield no advertising revenue, the platforms only compete for exclusivity to gain monopoly power on the advertising side. Multi-homing makes content investments less profitable for the platforms. Thus, an increase in the intensity of multi-homing implies worse equilibrium content.

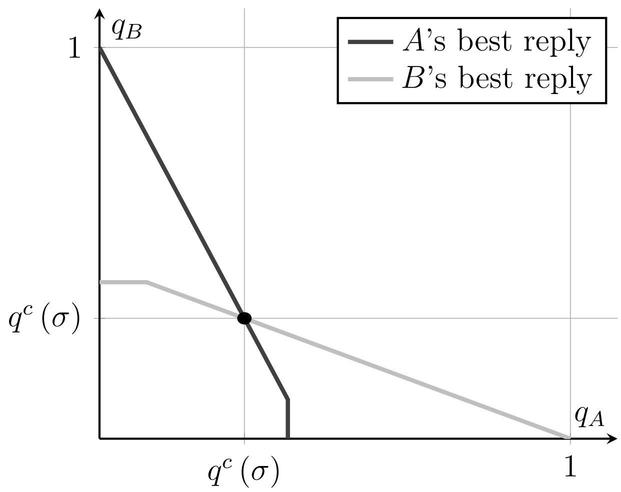

Figure 7: Competitive equilibrium with multi-homing.

Put differently, multi-homing shifts competition from the consumer side to the advertising side of the market. At $\sigma = 0$, no consumers are shared and the platforms have a monopoly over their own audiences. At the other extreme, $\sigma = 1$, there is no competition for the audiences and the platforms choose the monopoly contents, but competition for ad money is most intense.

4.4 Ad sales representation

Let us now consider that the platforms cooperate on selling their ads and that cooperation takes the form of a representation agreement (w, f), as described in the baseline model. As a representative of both platforms, A can sell the advertising slots at the monopoly price. As before, B obtains a fixed fee f and a wholesale price w from each consumer it attracts: $T(N_B) = f + wN_B$ is the tariff paid by A. Making use of the expressions derived in Remark 2 above, we obtain:

$$\frac{\partial N_B}{\partial q_A}(q_B, q_A; \sigma) = -\min\{q_A, (1-\sigma) q_B\},$$
$$\frac{\partial N_B}{\partial q_B}(q_B, q_A; \sigma) = 1 - q_A + \sigma q_A + (1-\sigma)\min\{q_A, (1-\sigma) q_B\}.$$

We have the following generalization of Lemma 1:

Lemma 5. *For any agreement (w, f) with $w \in [0, \rho]$ there exists a unique and stable continuation equilibrium $(q_A(w,\sigma), q_B(w,\sigma))$ such that:*

$$c'(q_A(w,\sigma)) = \rho - [\rho - w(1-\sigma)]\, q_B(w,\sigma),$$
$$c'(q_B(w,\sigma)) = w - w(1-\sigma)[q_A(w,\sigma) - (1-\sigma) q_B(w,\sigma)],$$

with $(1-\sigma) q_B(w,\sigma) < q_A(w,\sigma)$ for any $w \in (0, \rho)$. In particular, $q_B(0,\sigma) = 0$ and $q_A(0,\sigma) = q^c$.

It is again useful to think about the marginal advertising revenues at symmetric contents q. By improving its content slightly, A increases its revenue by $\rho(1-q) + w(1-\sigma) q$, where $1-q$ is the number of new consumers and $(1-\sigma) q$ the number of consumers who switch from B to single-home on A, reducing the payment. B's marginal revenue is instead

$$w(1-q) + w(1-\sigma) q + w\sigma^2 q,$$

where $1-q$ is due to market expansion, $(1-\sigma) q$ captures switchers from A, whereas $\sigma^2 q$ stands for previous single-homers on A who decide to multi-home due to B's content improvement.

Multi-homing makes A less responsive to changes in w, because increasing its own content steals fewer consumers from B (who gets the same wholesale price also from shared consumers). In particular, when the values are independent, there is no competition for the audiences and A no longer responds to the wholesale price, as it cannot affect the payment. Having no incentive to distort its own content, A chooses it to maximize the total advertising revenue given the content chosen by B.

We have the following generalization of Proposition 1:

Proposition 2. *The optimal contract* $(w^*(\sigma), f)$ *increases the price of advertising to the monopoly level, harming the advertisers, and yields contents such that*

$$q_B(w^*(\sigma), \sigma) \leq q^m \leq q_A(w^*(\sigma), \sigma),$$

where the inequalities are strict for any $\sigma < 1$. *In particular, with independent values,* $w^*(1) = \rho(1 - q^m)$ *implements the monopoly outcome. In this case the fixed fee that splits the industry-wide monopoly profit in half is given by*

$$f^* = \rho\left(1 - \frac{q^m}{2}\right)(q^m)^2 > 0.$$

Proposition 2 shows that the main insights from the baseline model carry over to a more general framework with multi-homing consumers. Cooperation enables the platforms to eliminate competition for the advertisers, leading to the monopoly price. Furthermore, unless the consumer valuations are independent, the optimal contract implements an asymmetric equilibrium, where the principal still under-invests and the sales agent over-invests compared to the monopoly benchmark. A difference to the baseline model is, however, that for certain values of σ the sales agent may overshoot and invest more than in the competitive benchmark. Simulations with quadratic costs reveal that overall consumers surplus nevertheless goes down.

5 Extensions

5.1 Augmented two-part tariffs

We now allow the platforms to augment the two part tariff with a payment that is linear in A's exclusive audience. As it turns out, this rather simple modification of the payment rule is sufficient for the platforms to be able to implement the monopoly outcome for any degree of redundancy in contents:

Proposition 3. *For any $\sigma \in [0,1]$ the following tariff implements the monopoly outcome in a unique and stable continuation equilibrium:*

$$T\left(N_B, \hat{N}_A\right) = f + w^m\left(\sigma\right) N_B + \hat{w}^m\left(\sigma\right) \hat{N}_A,$$

where:

$$\begin{aligned} w^m\left(\sigma\right) &= \frac{\rho\left(1-\sigma q^m\right)}{1+\left(1-\sigma\right)^2 q^m}, \\ \hat{w}^m\left(\sigma\right) &= \frac{\rho\left(1-\sigma\right) q^m}{1+\left(1-\sigma\right)^2 q^m} < w^m\left(\sigma\right). \end{aligned}$$

The monopoly outcome can thus be achieved by augmenting the two-part tariff with another wholesale price that is paid for each single-homer on A. This wholesale price is smaller than the one paid for B's audience. In particular, it is zero when the values are independent:

$$\begin{aligned} w^m\left(1\right) &= \rho\left(1-q^m\right) = w^*\left(1\right), \\ \hat{w}^m\left(1\right) &= 0. \end{aligned}$$

In other words, Proposition 3 corresponds to Proposition 2 if $\sigma = 1$. As described above, in this case A has the same incentives as an industry-wide monopoly, because its own choice of content does not affect B's overall number of consumers, but only increases the share of multi-homers.

However, even a slight degree of redundancy in contents creates an incentive for A to distort its own content upwards, as part of the consumers on B will then become single-homers on A, reducing the payment. The more general tariff restores the correct incentives by charging a wholesale price to A for its single-homers. This new wholesale price is decreasing in σ, which inversely measures the intensity of competition for the audiences. In particular, in the baseline model with no multi-homing, the platforms obtain the monopoly outcome by setting

$$
\begin{aligned}
w^m(0) &= \frac{\rho}{1+q^m}, \\
\hat{w}^m(0) &= \frac{\rho q^m}{1+q^m}.
\end{aligned}
$$

Remark 3. Even if the platforms do not possess a technology that sorts out multi-homers from single-homers, they can still implement the optimal tariff using the fact that the number of single-homers on A equals the total audience less B's audience.

5.2 Joint ownership

So far, we have studied the implications of a representation agreement on content investments. However, this is not the only way to cooperate on advertising sales. Another possibility is to set up a joint ad sales house and share the resulting joint advertising revenue ρN according to some ownership structure. To examine the effects of such a deal, suppose that A obtains a share $s \in [0,1]$ and B a share $1-s$ of the total advertising revenue. Suppose, further, that the deal may include a fixed fee f, allowing us to focus on the jointly optimal ownership structure.

We make the following assumption:

Assumption 2. *The ratio* $c''(q)/[c'(q)]^2$ *is non-decreasing in* q *and* $c''(0) > \rho$.

This ensures the existence and uniqueness of equilibria, and that choosing equal shares uniquely maximizes the joint profit.

For any joint venture (s, f) the platforms choose their contents simultaneously and independently to maximize their own profits. The best responses are given by

$$c'(R_i(q_j)) = \rho(1 - q_j)s_i, \tag{3}$$

where we have adopted the notation $s_A = s$ and $s_B = 1 - s$. We have the following result:

Proposition 4. *Equal split is the uniquely optimal joint venture. As a result, the platforms choose symmetric contents* q^s *such that*

$$c'(q^s) = \frac{\rho}{2}(1 - q^s),$$

implying that content investment is reduced below the monopoly level: $q^s < q^m$.

By setting up a joint ad sales house and sharing the total advertising revenue the platforms modify their investment incentives. In fact, this form of cooperation makes it possible for each platform to benefit from the investments taken by the other one. Without full ownership the platforms will thus only have partial appropriability of their investments, and this reduces their incentives to invest. There is a trade-off between promoting investment from the parties, as increasing the share of one necessarily decreases that of the other.

The optimal joint venture has equal shares and the platforms will invest less than the monopoly. This because of partial appropriability of the investments: half of the revenue generated by the investment goes to the other party, and not taking this positive externality into account, the platforms will under-invest in equilibrium. Figure 8 illustrates this graphically.

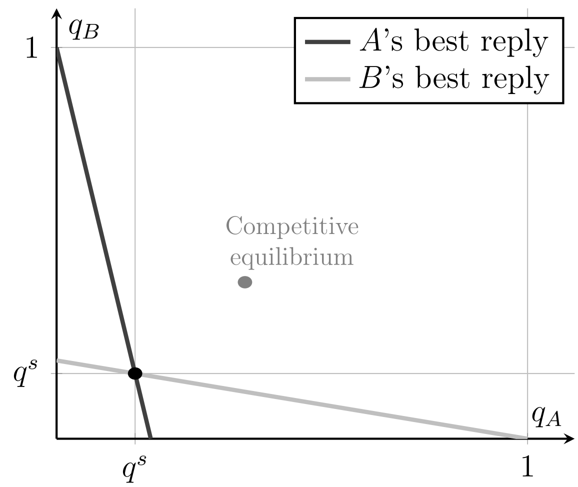

Figure 8: Cooperative equilibrium for $s = 1/2$.

6 Conclusion

This paper analyzes the effects of advertising sales' cooperation on content provision. Cooperation eliminates competition for the advertisers and increases the per consumer advertising revenue. This, however, does not lead to more content investment, because cooperation also makes business stealing less profitable. We show that this latter effect dominates, and as a result, cooperation on advertising harms not only advertisers, who end up paying more, but also consumers of content.

The effect on the consumer side depends on the form of cooperation. An advertising sales representation agreement, where the sales agent pays the principal a wholesale price for its customers, leads to an asymmetric equilibrium: the principal invests less and the sales agent more than an industry-wide monopoly. Hence, the consumers on the principal's platform are most harmed. This result carries over to multi-homing consumers, as long as there is some degree of redundancy in content provision. In the particular case of independent values, the platforms choose symmetric contents that maximize the industry profit.

This monopoly outcome can be achieved more generally through a more sophisticated representation agreement, whereby the sales agent also pays the principal a price for the number of single-homers on the sales agent's own platform. By setting this price below the wholesale price paid for the principal's customers, the platforms are able to induce monopoly content from both. By contrast, a joint advertising sales house, where the platforms simply split the total advertising revenue, fails to do this, due to free-riding. As a result, content provision falls below the monopoly level on both platforms.

The analysis suggests a cautious attitude towards claims that cooperation on advertising, and resulting higher ad prices, should be allowed, because consumers will benefit from better content provision. Although it is true that an increase in advertising revenue makes content investments more profitable, the payment between the parties affects the appropriability of these investments and may lessen competition for the audiences.

There are at least two interesting avenues for future research. First, we have assumed that consumers have a neutral attitude towards advertising. In practice, however, some consumers dislike advertising; an extension of our model would help us understanding how cooperation on advertising affects not only the choice of content, but also the number of ads shown to a consumer. Second, we have assumed that content is offered free of charge. Adding subscription fees to the model would shed light on the effects of ad cooperation on the pricing of content.

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Appendix

Proof of Lemma 4

Set $\sigma = 0$ to obtain the competitive equilibrium in the baseline model.

Proof. Let us characterize the best response, $R_i(q_j)$, for any $q_j \geq 0$. The marginal profit,

$$\frac{\partial \pi_i}{\partial q_i}(q_i, q_j) = -c'(q_i) + \rho(1 - q_j) + \rho \min\{q_j, (1-\sigma) q_i\},$$

is strictly decreasing in q_i for any $q_j \geq 0$, by Assumption 1. Together with $c'(0) = 0$ Assumption 1 also implies $c'(1) > 2\rho$ and therefore

$$\frac{\partial \pi_i}{\partial q_i}(1; q_j) \leq -c'(1) + 2\rho < 0.$$

Hence, for any $q_j \geq 0$, the first-order condition $\frac{\partial \pi_i}{\partial q_i} = 0$ is necessary and sufficient for optimality, characterizing the best response:

$$c'(R_i(q_j)) = \rho(1 - q_j) + \rho \min\{q_j, (1-\sigma) R_i(q_j)\}.$$

Using the implicit function theorem, $R_i'(q_j)$ equals zero if $R_i(q_j) > \frac{q_j}{1-\sigma}$. If instead $R_i(q_j) < \frac{q_j}{1-\sigma}$, then

$$R_i'(q_j) = -\frac{\rho}{c''(R_i(q_j)) - \rho(1-\sigma)},$$

where $c''(R_i(q_j)) > 2\rho$ by Assumption 1 and therefore

$$c''(R_i(q_j)) - \rho(1-\sigma) > (1+\sigma)\rho \geq \rho.$$

Thus, $R_i'(q_j)$ is at most zero and strictly above -1. By symmetry there exists a unique and stable, symmetric equilibrium, characterized by

$$c'\left(q^c\left(\sigma\right)\right) = \rho\left(1 - \sigma q^c\left(\sigma\right)\right).$$

Applying the implicit function theorem, we obtain

$$\left(q^c\right)'\left(\sigma\right) = \frac{\rho q^c\left(\sigma\right)}{c''\left(q^c\left(\sigma\right)\right) + \rho\sigma} > 0,$$

by convexity of the cost function. Finally, $c'\left(q^c\left(1\right)\right) = \rho\left[1 - q^c\left(1\right)\right]$ is the same condition as in the monopoly benchmark, so $q^c\left(1\right) = q^m$. □

Proof of Lemma 5

Set $\sigma = 0$ to obtain Lemma 1.

Proof. Fix any $w \in [0, \rho]$. Let us characterize the best responses, $R_A\left(q_B\right)$ and $R_B\left(q_A\right)$, for any $q_A, q_B \geq 0$. The first-order conditions are:

$$\frac{\partial \pi_A}{\partial q_A}\left(q_A, q_B\right) = -c'\left(q_A\right) + \rho\left(1 - q_B\right) + w \min\left\{q_A, \left(1-\sigma\right) q_B\right\} = 0,$$
$$\frac{\partial \pi_B}{\partial q_B}\left(q_A, q_B\right) = -c'\left(q_B\right) + w\left(1 - q_A\right) + w\sigma q_A + w\left(1-\sigma\right) \min\left\{q_A, \left(1-\sigma\right) q_B\right\} = 0.$$

The marginal profits are strictly decreasing in own content, as Assumption 1 and $\rho \geq w$ imply $c''\left(q_i\right) > w$. Together with $c'\left(0\right) = 0$ Assumption 1 also implies $c'\left(1\right) > 2\rho$ and therefore

$$\frac{\partial \pi_i}{\partial q_i}\left(1, q_j\right) \leq -c'\left(1\right) + \rho + w < 0 \text{ for each } i.$$

Thus, $R_A(q_B) < 1$ and $R_B(q_A) < 1$, and satisfy the first-order conditions:

$$c'(R_A(q_B)) = \rho(1-q_B) + w\min\{R_A(q_B), (1-\sigma)q_B\},$$
$$c'(R_B(q_A)) = w(1-q_A) + w\sigma q_A + w(1-\sigma)\min\{q_A, (1-\sigma)R_B(q_A)\}.$$

Using the implicit function theorem, we have:

$$R'_A(q_B) = \begin{cases} -\frac{\rho-(1-\sigma)w}{c''(R_A(q_B))} & \text{if } (1-\sigma)q_B < R_A(q_B), \\ -\frac{\rho}{c''(R_A(q_B))-w} & \text{if } (1-\sigma)q_B > R_A(q_B), \end{cases}$$

and

$$R'_B(q_A) = \begin{cases} 0 & \text{if } q_A < (1-\sigma)R_B(q_A), \\ -\frac{(1-\sigma)w}{c''(R_B(q_A))-w(1-\sigma)^2} & \text{if } q_A > (1-\sigma)R_B(q_A). \end{cases}$$

By Assumption 1, $c''(R_i(q_j)) > 2\rho$, implying that $R'_i(q_j)$ is at most zero and strictly above -1 for each i. Hence, if there exists an equilibrium, then it is unique and stable. For existence, observe that $R_A(1) = 0$ and $R_B(0) \leq q^c$ together imply $R_B(R_A(1)) \leq q^c < 1$. Furthermore, $R_A(0) = q^c$ and $R_B(q^c) > 0$ imply $R_B(R_A(0)) > 0$. By continuity there thus exists $q > 0$ such that $R_B(R_A(q)) = q$, ensuring existence.

Let us show that in equilibrium $(1-\sigma)q_B < q_A$ for any $w < \rho$. Suppose, by contradiction, that $(1-\sigma)q_B \geq q_A$. Then, the equilibrium conditions imply:

$$c'(q_A) = \rho(1-q_B) + wq_A,$$
$$c'(q_B) = w.$$

By Assumption 1:

$$\begin{aligned}2\rho\left(q_B - q_A\right) &\leq c'\left(q_B\right) - c'\left(q_A\right)\\ &= w - \rho\left(1 - q_B\right) - wq_A\\ &< \rho\left(q_B - q_A\right),\end{aligned}$$

which is a contradiction. Therefore, $w < \rho$ implies $\left(1-\sigma\right) q_B < q_A$. $\square$

Proof of Lemma 2

Let us prove the comparative statics for any σ as the general expressions will be useful in the proof of Proposition 2 below.

Proof. Consider any $w \in (0, \rho)$. Using Lemma 5 and the implicit function theorem:

$$\begin{aligned}c''\left(q_A\right)\frac{\partial q_A}{\partial w} &= \left(1-\sigma\right) q_B - \left[\rho - w\left(1-\sigma\right)\right]\frac{\partial q_B}{\partial w},\\ c''\left(q_B\right)\frac{\partial q_B}{\partial w} &= 1 - \left(1-\sigma\right) q_A - \left(1-\sigma\right)^2 q_B - w\left(1-\sigma\right)\frac{\partial q_A}{\partial w} + w\left(1-\sigma\right)^2\frac{\partial q_B}{\partial w},\end{aligned}$$

where $q_i = q_i\left(w,\sigma\right)$ to shorten notation. These solve for:

$$\begin{aligned}\frac{\partial q_A}{\partial w} &= \frac{\left(1-\sigma\right) q_B\left[c''\left(q_B\right) - \rho\left(1-\sigma\right)\right] - \left[\rho - w\left(1-\sigma\right)\right]\left[1 - \left(1-\sigma\right) q_A\right]}{H},\\ \frac{\partial q_B}{\partial w} &= \frac{\left[1 - \left(1-\sigma\right) q_A\right] c''\left(q_A\right) + q_B\left[c''\left(q_A\right) - w\right]\left(1-\sigma\right)^2}{H},\end{aligned}$$

where

$$H = c''\left(q_A\right)\left[c''\left(q_B\right) - \rho\left(1-\sigma\right)\right] + \left(1-\sigma\right)\left[\rho - w\left(1-\sigma\right)\right]\left[c''\left(q_A\right) - w\right].$$

By Assumption 1, $c''\left(q_B\right) > \rho\left(1-\sigma\right)$ and $c''\left(q_A\right) > w$, so the Hessian is strictly positive:

$H > 0$. By Lemma 5, $q_A(w,\sigma) \leq q^c < 1$. Therefore, $\frac{\partial q_B}{\partial w} > 0$ and $\frac{\partial q_A}{\partial w} + \frac{\partial q_B}{\partial w} > 0$. Furthermore, $\frac{\partial q_A}{\partial w} < 0$ for $w > 0$ sufficiently close to zero. □

Proof of Proposition 1 and 2

Proof. Let $\Pi(w)$ denote the joint platform profit in the continuation equilibrium for any given $w \in [0,\rho]$. By the envelope theorem:

$$\begin{aligned}
\Pi'(w) &= \frac{\partial \pi_A}{\partial q_B}\frac{\partial q_B}{\partial w} + \frac{\partial \pi_B}{\partial q_A}\frac{\partial q_A}{\partial w} + \overbrace{\frac{\partial \pi_A}{\partial w} + \frac{\partial \pi_B}{\partial w}}^{=0} \\
&= (\rho - w)(1 - q_A)\frac{\partial q_B}{\partial w} - w\sigma q_A \frac{\partial q_B}{\partial w} - w(1-\sigma) q_B \left[\frac{\partial q_A}{\partial w} + (1-\sigma)\frac{\partial q_B}{\partial w}\right],
\end{aligned}$$

where $\frac{\partial q_B}{\partial w} > 0$ and $\frac{\partial q_A}{\partial w} + \frac{\partial q_B}{\partial w} > 0$ were shown in the proof of Lemma 2 above. Consequently, $\Pi'(0) > 0$ and

$$\begin{aligned}
\Pi'(\rho) &= -\rho\sigma q_A \frac{\partial q_B}{\partial w} - \rho(1-\sigma) q_B \left[\frac{\partial q_A}{\partial w} + (1-\sigma)\frac{\partial q_B}{\partial w}\right] \\
&\leq -\rho\sigma(1-\sigma) q_B \frac{\partial q_B}{\partial w} - \rho(1-\sigma) q_B \left[\frac{\partial q_A}{\partial w} + (1-\sigma)\frac{\partial q_B}{\partial w}\right] \\
&= -\rho(1-\sigma) q_B \left[\frac{\partial q_A}{\partial w} + \frac{\partial q_B}{\partial w}\right] \\
&< 0,
\end{aligned}$$

where the second inequality uses $q_A > (1-\sigma) q_B$ by Lemma 5. By Bolzano's theorem, there exists $w^* \in (0,\rho)$ such that $\Pi'(w^*) = 0$, which is necessary for optimality.

It remains to show the properties of the equilibrium. To shorten notation, denote $q_i^* = q_i(w^*,\sigma)$. Let us first show that $q_B^* \leq q^m$ holds. Suppose, by contradiction, that $q_B^* > q^m$ is the case. On the one hand, by B's revealed preference, we have

$$\pi_B(q_A^*, q_B^*) - \pi_B(q_A^*, q^m) = wN_B(q_A^*, q_B^*) - wN_B(q_A^*, q^m) + c(q^m) - c(q_B^*) \geq 0,$$

where

$$N_B\left(q_A^*, q_B^*\right) - N_B\left(q_A^*, q^m\right) = \left(q^m - q_B^*\right)\left[1 - (1-\sigma)\,q_A^* + \frac{(1-\sigma)^2}{2}\left(q^m + q_B^*\right)\right].$$

On the other hand, by the monopoly's revealed preference,

$$\Pi\left(q^m, q^m\right) - \Pi\left(q^m, q_B^*\right) = \rho\left(1 - q^m\right)\left(q^m - q_B^*\right) + c\left(q_B^*\right) - c\left(q^m\right) \geq 0.$$

Combining these inequalities, we obtain

$$\left[w - w(1-\sigma)\,q_A^* + \frac{w(1-\sigma)^2}{2}\left(q^m + q_B^*\right) + \rho\left(1 - q^m\right)\right]\left(q_B^* - q^m\right) \leq 0,$$

which after using the first-order conditions becomes

$$\left[c'\left(q^m\right) + c'\left(q_B^*\right) - \frac{w(1-\sigma)^2}{2}\left(q_B^* - q^m\right)\right]\left(q_B^* - q^m\right) \leq 0.$$

Then, taking the difference $q_B^* - q^m > 0$ as a common factor, we have a contradiction:

$$\begin{aligned}
0 &\geq \left[\frac{2c'\left(q^m\right)}{q_B^* - q^m} + \frac{c'\left(q_B^*\right) - c'\left(q^m\right)}{q_B^* - q^m} - \frac{w(1-\sigma)^2}{2}\right]\left(q_B^* - q^m\right)^2 \\
&> \left[\frac{2c'\left(q^m\right)}{q_B^* - q^m} + 2\rho - \frac{w(1-\sigma)^2}{2}\right]\left(q_B^* - q^m\right)^2 \\
&> 0,
\end{aligned}$$

where the second inequality follows from Assumption 1. Thus, we conclude that $q_B^* \leq q^m$. It follows directly from Lemma 5 that $q_A^* < q^c$.

In particular, for $\sigma = 1$ the optimality condition writes

$$\Pi'\left(w^*\right) = \left[\left(\rho - w^*\right)\left(1 - q_A^*\right) - w^* q_A^*\right]\frac{\partial q_B}{\partial w} = 0 \Longrightarrow w^* = \rho\left(1 - q_A^*\right).$$

Lemma 5 implies that $c'\left(q_B^*\right) = w^* = \rho\left(1 - q_A^*\right)$ and $c'\left(q_A^*\right) = \rho\left(1 - q_B^*\right)$, and therefore $q_A^* = q_B^* = q^m$. For $\sigma < 1$ instead $q_B^* \leq q^m$ implies that

$$c'\left(q^m\right) = \rho\left(1 - q^m\right) < \rho - \left(\rho - w^*\left(1 - \sigma\right)\right) q_B^* = c'\left(q_A^*\right),$$

and therefore $q^m < q_A^*$ by convexity of costs.

Finally, returning to the case $\sigma = 1$, the fixed fee f^* that splits the monopoly profit solves

$$\rho\left(2 - q^m\right) q^m - w^*\left(1\right) N_B\left(q^m, q^m\right) - f^* = w^*\left(1\right) N_B\left(q^m, q^m\right) + f^*,$$

and therefore

$$\begin{aligned} f^* &= \rho\left(2 - q^m\right) \frac{q^m}{2} - w^*\left(1\right) N_B\left(q^m, q^m\right) \\ &= \rho\left(2 - q^m\right) \frac{q^m}{2} - \rho\left(1 - q^m\right) q^m + \rho\left(1 - q^m\right) \frac{\left(q^m\right)^2}{2} \\ &= \rho\left(1 - \frac{q^m}{2}\right) \left(q^m\right)^2. \end{aligned}$$

□

Proof of Proposition 3

Proof. Suppose the platforms agree on the following tariff:

$$T\left(N_B, \hat{N}_A\right) = f + w^m\left(\sigma\right) N_B + \hat{w}^m\left(\sigma\right) \hat{N}_A,$$

where:

$$w^m(\sigma) = \frac{\rho(1-\sigma q^m)}{1+(1-\sigma)^2 q^m},$$
$$\hat{w}^m(\sigma) = \frac{\rho(1-\sigma)q^m}{1+(1-\sigma)^2 q^m} < w^m(\sigma).$$

Let us first show that the equilibrium is unique, interior and stable. The first-order conditions are:

$$\frac{\partial \pi_A}{\partial q_A}(q_A, q_B) = -c'(q_A) + \rho(1-q_B) - \frac{\partial T}{\partial q_A},$$
$$\frac{\partial \pi_B}{\partial q_B}(q_A, q_B) = -c'(q_B) + \frac{\partial T}{\partial q_B},$$

where

$$\frac{\partial T}{\partial q_A} = \hat{w}^m(1-q_B) - (w^m - \hat{w}^m)\min\{q_A, (1-\sigma)q_B\},$$
$$\frac{\partial T}{\partial q_B} = w^m(1-q_A) + (w^m - \hat{w}^m)[\sigma q_A + (1-\sigma)\min\{q_A, (1-\sigma)q_B\}].$$

The second derivatives are:

$$\frac{\partial^2 \pi_A}{\partial q_A^2}(q_A, q_B) = -c''(q_A) + \begin{cases} w^m - \hat{w}^m & \text{if } q_A < (1-\sigma)q_B, \\ 0 & \text{if } q_A > (1-\sigma)q_B, \end{cases}$$

and

$$\frac{\partial^2 \pi_A}{\partial q_B^2}(q_A, q_B) = -c''(q_B) + \begin{cases} 0 & \text{if } q_A < (1-\sigma)q_B, \\ (w^m - \hat{w}^m)(1-\sigma)^2 & \text{if } q_A > (1-\sigma)q_B. \end{cases}$$

By Assumption 1 and $\rho > w^m$ these are negative. Together with $c'(0) = 0$ Assumption 1 also implies $c'(1) > 2\rho$ and therefore

$$\frac{\partial \pi_i}{\partial q_i}(1, q_j) \leq -c'(1) + 2\rho < 0 \text{ for each } i.$$

Thus, $R_A(q_B) < 1$ and $R_B(q_A) < 1$, and satisfy the first-order conditions:

$$c'(R_A(q_B)) = (\rho - \hat{w}^m)(1 - q_B) + (w^m - \hat{w}^m)\min\{R_A(q_B), (1-\sigma) q_B\},$$
$$c'(R_B(q_A)) = w^m(1 - q_A) + (w^m - \hat{w}^m)[\sigma q_A + (1-\sigma)\min\{q_A, (1-\sigma) R_B(q_A)\}].$$

Using the implicit function theorem, we have:

$$R'_A(q_B) = \begin{cases} -\frac{\rho - \hat{w}^m - (1-\sigma)(w^m - \hat{w}^m)}{c''(R_A(q_B))} & \text{if } (1-\sigma) q_B < R_A(q_B), \\ -\frac{\rho - \hat{w}^m}{c''(R_A(q_B)) - (w^m - \hat{w}^m)} & \text{if } (1-\sigma) q_B > R_A(q_B), \end{cases}$$

where

$$\begin{aligned} \rho &> \hat{w}^m + (1-\sigma)(w^m - \hat{w}^m) \\ &= \frac{\rho(1-\sigma)}{1 + (1-\sigma)^2 q^m}. \end{aligned}$$

Furthermore:

$$R'_B(q_A) = \begin{cases} -\frac{\hat{w}^m}{c''(R_B(q_A))} & \text{if } q_A < (1-\sigma) R_B(q_A), \\ -\frac{(1-\sigma) w^m + \sigma \hat{w}^m}{c''(R_B(q_A)) - (w^m - \hat{w}^m)(1-\sigma)^2} & \text{if } q_A > (1-\sigma) R_B(q_A). \end{cases}$$

By Assumption 1, $c''(R_i(q_j)) > 2\rho$, implying that $R'_i(q_j)$ is at most zero and strictly above -1 for each i. Hence, if there exists an equilibrium, then it is unique and stable.

It remains to show that $R_A(q^m) = R_B(q^m) = q^m$. Indeed:

$$
\begin{aligned}
c'(R_A(q^m)) &= (\rho - \hat{w}^m)(1 - q^m) + (w^m - \hat{w}^m)\min\{R_A(q^m), (1-\sigma)q^m\} \\
&= \rho(1 - q^m) + (w^m - \hat{w}^m)\min\{R_A(q^m), (1-\sigma)q^m\} - \hat{w}^m(1 - q^m), \\
&= \rho(1 - q^m) + \rho(1 - q^m)\frac{\min\{R_A(q^m), (1-\sigma)q^m\} - (1-\sigma)q^m}{1 + (1-\sigma)^2 q^m} \\
&= \rho(1 - q^m) \\
&= c'(q^m),
\end{aligned}
$$

and

$$
\begin{aligned}
c'(R_B(q^m)) &= w^m(1 - q^m) + (w^m - \hat{w}^m)\sigma q^m + (w^m - \hat{w}^m)(1-\sigma)\min\{q^m, (1-\sigma)R_B(q^m)\} \\
&= \rho(1 - q^m)\frac{1 + (1-\sigma)\min\{q^m, (1-\sigma)R_B(q^m)\}}{1 + (1-\sigma)^2 q^m} \\
&= \rho(1 - q^m) \\
&= c'(q^m).
\end{aligned}
$$

□

Proof of Proposition 4

Proof. Let us first show that, for any joint venture (s, f), there exists a unique and stable continuation equilibrium with contents$(q_A(s), q_B(s))$ satisfying:

$$
\begin{aligned}
c'(q_A(s)) &= \rho[1 - q_B(s)]s, \\
c'(q_B(s)) &= \rho[1 - q_A(s)](1 - s).
\end{aligned}
$$

Indeed, the first-order condition for i is

$$c'(q_i) = \rho \frac{\partial N(q_A, q_B)}{\partial q_i} s_i = \rho [1 - q_j] s_i.$$

By Assumption 2, this equation has a unique solution $q_i = R_i(q_j) < 1$ (the best reply of platform i) for any $q_j \geq 0$. Furthermore, by the implicit function theorem $R_i'(q_j) = -\rho s_i / c''(R_i(q_j))$, which is at most zero and strictly above -1 for $s_i < 1$ by Assumption 2, implying the existence, stability and uniqueness of equilibria (at $s_i = 1$ the other platform shuts down).

Using the implicit function theorem:

$$\begin{aligned} c''(q_A(s)) q_A'(s) &= \rho [1 - q_B(s) - q_B'(s) s], \\ c''(q_B(s)) q_B'(s) &= -\rho [1 - q_A(s) + q_A'(s)(1 - s)]. \end{aligned}$$

These solve for:

$$\begin{aligned} q_A'(s) &= \frac{\rho [1 - q_B(s)] c''(q_B(s)) + \rho^2 [1 - q_A(s)] s}{H(s)}, \\ q_B'(s) &= -\frac{\rho [1 - q_A(s)] c''(q_A(s)) + \rho^2 [1 - q_B(s)] (1 - s)}{H(s)}, \end{aligned}$$

where

$$H(s) = c''(q_A(s)) c''(q_B(s)) - s(1 - s) \rho^2.$$

By Assumption 1 we have $H(s) > 0$, $q_A'(s) > 0$ and $q_B'(s) < 0$.

Let $\Pi(s)$ denote the joint platform profit in the continuation equilibrium for given

$s \in [0,1]$. By the envelope theorem:

$$\Pi'(s) = \frac{\partial \pi_A}{\partial q_B} q_B'(s) + \frac{\partial \pi_B}{\partial q_A} q_A'(s) + \overbrace{\frac{\partial \pi_A}{\partial s} + \frac{\partial \pi_B}{\partial s}}^{=0}$$
$$= s\left[1 - q_A(s)\right] q_B'(s) + (1-s)\left[1 - q_B(s)\right] q_A'(s),$$

where $q_A'(s) > 0 > q_B'(s)$. Consequently, $\Pi'(0) > 0 > \Pi'(1)$. In particular, $\Pi'\left(\frac{1}{2}\right) = 0$. It remains to show that the equal split is uniquely optimal. Using the expressions for $q_B'(s)$ and $q_A'(s)$ the optimality condition $\Pi'(s) = 0$ can be written as

$$(1-s)^3 \frac{c''(q_B(s))}{\left[c'(q_B(s))\right]^2} = s^3 \frac{c''(q_A(s))}{\left[c'(q_A(s))\right]^2}.$$

As $q_A'(s) > 0 > q_B'(s)$, by Assumption 2 the left-hand side is strictly decreasing in s, whereas the right-hand side is strictly increasing in s. Thus, the equal split is uniquely optimal.

It remains to show that $q^s < q^m$. Suppose, by contradiction, that $q^s \geq q^m$. Then, by convexity of costs:

$$\frac{\rho}{2}\left[1 - q^s\right] = c'(q^s) \geq c'(q^m) = \rho\left[1 - q^m\right],$$

which implies that $2q^m \geq 1 + q^s$. Together with $q^s \geq q^m$ this implies $q^m \geq 1$, which is a contradiction. $\square$