Inverse Pricing in Two-Sided Markets (Preliminary)

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Abstract

This article derives a robust pricing mechanism for two-sided markets, when users on each side privately know their network benefits. The mechanism is optimal in the class of Bayesian incentive compatible and interim individually rational mechanisms, allowing the platform to do better than with posted prices (Rochet and Tirole, 2003). The mechanism is robust in the sense of dominant strategy incentive compatibility and ex post individual rationality.

1 Introduction

Two-sided markets are prone to coordination failures, due to network externalities between users from different sides of the market: in credit card markets, cardholders care about the number of merchants accepting card payments with their card, but so do merchants care about the number of customers carrying that card (Rochet and Tirole, 2003). Therefore beliefs matter: if merchants believe only few customers are carrying the card, they have little incentives to join the payment platform and not many stores will accept payments with the card. Then customers do not benefit from carrying the card and the pessimistic beliefs constitute an equilibrium. The market may fail entirely. Similarly, optimistic beliefs can make the platform work well so that in equilibrium both sides of the market come on board.

The problem of multiplicity in equilibria is well known in the literature on two-sided markets (Caillaud and Jullien, 2003; Weyl, 2010). This article derives a robust platform pricing mechanism, when users on each side of the market privately know how much they benefit from joining the platform, given the number of users on the other sides of the market. This mechanism is called inverse pricing and it solves the coordination failure, being robust to different beliefs held by the users. Inverse pricing is dominant strategy incentive compatible, ex post individually rational and ex post budget balanced (Hagerty and Rogerson, 1987). Furthermore, inverse pricing is optimal in the class of Bayesian incentive compatible and individually rational mechanisms, in particular yielding higher welfare and platform profit than posted prices in Rochet and Tirole (2003). This means that there is no loss for platforms to use inverse pricing instead of more complex pricing schemes that depend on various beliefs held by the users.

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The idea behind inverse pricing is the following. A user with a high network benefit is matched to more users on the other side of the market, because the price charged from users that interact with this user is set low. By contrast, a user with a low network benefit is matched to a smaller number of users on the other side of the market, because the price charged from that side is high. Therefore users with high network benefits are matched with both high and low type users, whereas the low type users are only matched with high type users and therefore have to pay a low price for a small number of interactions. The high type users are matched with a large number of users on the other side of the market, but they need to pay a high price when they interact with the low type users.

In its essence, inverse pricing translates the two-sided screening problem into a canonical, one-sided Mussa and Rosen (1978) screening problem. The platform optimally selects a price schedule for one side of the market and then the other side is offered the inverse of this price schedule. The profit-maximizing prices are monopoly prices, where the effective cost of interaction on one side equals the interaction cost less the virtual interaction benefit obtained by the user on the other side. The virtual interaction benefit equals the true interaction benefit less the cost of information. In particular, there is no distortion at the top: the price charged from interacting with the highest user type on the other side is simply the monopoly price, based on the cost of interaction less the highest interaction benefit.

This work relates to various strands of economic literature: two-sided markets (Rochet and Tirole, 2003, 2006; Caillaud and Jullien, 2003; Armstrong, 2006; Weyl, 2010), mechanism design in two-sided markets (Gomes, 2014; Gomes and Pavan, 2015), robust mechanism design (Hagerty and Rogerson, 1987; Mookherjee and Reichelstein, 1992; Manelli and Vincent, 2010; Gershkov et al., 2013), bilateral trade (Myerson and Satterthwaite, 1983; Hagerty and Rogerson, 1987) and screening (Mussa and Rosen, 1978). The contribution is to derive a robust pricing mechanism for two-sided markets.

2 Model

2.1 Setup

There is a monopoly platform and a two-sided market populated with a unit mass of potential users on each side $i = A, B$ of the market.[1] An individual user on side A has the network benefit $\theta_A N_B$ from being matched with N_B users on side B, where the user's interaction benefit θ_A is private information. Similarly, a user on side B has the network benefit $\theta_B N_A$ from being matched with N_A users on side A, where θ_B denotes her privately known interaction benefit. User beliefs are not restricted. The platform, who enables interactions between the two sides of the market, believes that the interaction benefits are distributed according to a density f_i and cdf F_i over $\Theta_i \subset \mathbb{R}$ on side $i = A, B$. We make the following standard assumption.

Assumption 1. *Assume that the virtual interaction benefit*

$$b_i(\theta_i) \equiv \theta_i - \frac{1 - F_i(\theta_i)}{f_i(\theta_i)}$$

[1]Examples of user pairs include the cardholder-merchant pair on a payment platform, the reader-advertiser pair on a newspaper platform, the passenger-driver pair on a transportation platform and the consumer-producer pair on a trading platform.

are is non-decreasing for both $i = A, B$.

From each interaction, the platform incurs a cost $c \geq 0$ and charges two separate prices: an interaction price $p_A \in \mathbb{R}$ from side A and an interaction price $p_B \in \mathbb{R}$ from side B. The price vector $p = (p_A, p_B)$ may depend on the type $\theta = (\theta_A, \theta_B)$ of the interaction. Formally, the pricing mechanism $p : \Theta \longrightarrow \mathbb{R}^2$ is a mapping defined over the set $\Theta = \Theta_A \times \Theta_B$ of all types of potential interactions. The mechanism is subject to incentive compatibility and individual rationality constraints. Incentive compatibility requires that the price schedule must induce truth-telling from each individual user, eliciting privately held information. Furthermore, the mechanism must be individually rational so no user gets less than her outside option. Outside options are uniformly set to zero.

2.2 Linear pricing

Consider the monopoly platform model in Rochet and Tirole (2003) as a benchmark, where the platform posts non-discriminatory interaction prices, without trying to elicit private information. Users have the net utilities

$$u_A = (\theta_A - p_A)\, N_B \quad \text{and} \quad u_B = (\theta_B - p_B)\, N_A$$

from joining the platform and interacting with the other side of the market. Therefore all users pairs with $\theta_A \geq p_A$ and $\theta_B \geq p_B$ interact with one another and the total volume of interactions is given by the multiplication of these users:

$$N_A N_B = [1 - F_A(p_A)]\,[1 - F_B(p_B)]$$

From each interaction, the platform obtains the total price $p_A + p_B$ and incurs the cost $c \geq 0$ of interaction. Multiplying this profit margin with the total volume of interactions yields the platform's profit

$$\pi = (p_A + p_B - c)\,[1 - F_A(p_A)]\,[1 - F_B(p_B)]$$

The necessary conditions for profit maximization are

$$p_A + p_B - c = \frac{p_A}{\eta_A} = \frac{p_B}{\eta_B}$$

where

$$\eta_i \equiv -p_i \frac{N_i'}{N_i}$$

is matching elasticity for side $i = A, B$. The matching elasticity for side A measures the percentage change in the number matches for an individual side B user when the price charged from side A is increased by one percent. Proposition 1 summarizes the profit-maximizing linear pricing mechanism in terms of these elasticities.

Proposition 1. *(Rochet and Tirole, 2003, 2006) The profit-maximizing linear pricing mechanism satisfies the Lerner formulae*

$$\frac{p_A - (c - p_B)}{p_A} = \frac{1}{\eta_A}$$
$$\frac{p_B - (c - p_A)}{p_B} = \frac{1}{\eta_B}$$

Compared to one-sided monopoly pricing, platform pricing differs in the sense that for each interaction the platform receives two prices instead of one. This changes the effective cost of interaction: the optimal monopoly price for side A is chosen based on the cost of interaction less the price received from side B. Similarly, the optimal monopoly price for side B is chosen based on the cost of interaction less the price charged from side A. Furthermore, optimal pricing satisfies the so-called see-saw condition: if the price charged from one side is decreased, then the price for the other side i should be increased. The price increase is equal to the pass-through rate

$$\rho_i \equiv \frac{1}{1 - [p_i/\eta_i]'}$$

on that side, where the ratio of price to elasticity measures the platform's market power. Consequently, if the market power decreases, the pass-through rate is less than one-to-one. If the market power increases, the pass-through rate is more than one-to-one.[2] Assumption 1 implies that pass-through rates are always positive, i.e. that market power cannot increase more than the price is increased. Furthermore, the two-sided contraction $\rho_A \rho_B < 1$ condition ensures sufficiency Weyl (2010). The optimal price structure equalizes the platform's market power over the two sides:

$$\frac{p_A}{\eta_A} = \frac{p_B}{\eta_B}$$

In other words, the profit-maximizing linear prices are such that the price ratio equals the ratio of the matching elasticities.

2.3 Inverse pricing

Let us now consider prices that make use of private information and thus potentially discriminate between different types of potential interactions. In particular, we focus on the following pricing mechanism, referred to as inverse pricing.

Definition 1. An inverse pricing mechanism is defined as non-increasing functions $p_A : \Theta_B \longrightarrow \mathbb{R}$ and $p_B : \Theta_A \longrightarrow \mathbb{R}$ such that

$$p_A \circ p_B = \mathrm{id}_{\Theta_A}$$
$$p_B \circ p_A = \mathrm{id}_{\Theta_B}$$

[2] Decreasing (increasing) market power on side $i = A, B$ is equivalent to $1 - F_i$ being log-concave (log-convex). See Bagnoli and Bergstrom (2005).

That is, the platform asks each user to report her interaction benefit and then chooses the interaction price charged from the other side based on this report. The key feature of inverse pricing is that the interaction price paid by a user does not depend on her own report. By contrast, her interaction price depends on how much the other side benefits from the interaction: a higher interaction benefit for side B implies a lower interaction price for side A and vice versa. The following lemma shows that inverse pricing is dominant strategy incentive compatible and ex post individually rational.

Lemma 1. *Inverse pricing is dominant strategy incentive compatible and ex post individually rational.*

Proof. Consider an individual user on side A who reports an interaction benefit θ'_A to the platform. This report potentially differs from her true interaction benefit θ_A. The platform charges an interaction price $p_B(\theta'_A)$ from any user on side B who wishes to make an interaction with the user. The user has access to all side B users whose interaction benefit exceeds this price, amounting to the net utility

$$u_A(\theta'_A; \theta_A) = \int_{p_B(\theta'_A)}^{\bar{\theta}_B} [\theta_A - p_A(\theta_B)] \mu_B(\theta_B) \, d\theta_B$$

where $p_A(\theta_B)$ is the price charged from the user when it is matched with a type θ_B user on side B and μ_B is the user's belief on how the interaction benefits on side B are distributed. The optimal report θ'_A satisfies the necessary first-order condition

$$\frac{\partial u_A(\theta'_A; \theta_A)}{\partial \theta'_A} = -\dot{p}_B(\theta'_A) [\theta_A - p_A(p_B(\theta'_A))] \mu_B(p_B(\theta'_A)) = 0$$

In particular, the inverse relation $p_A(p_B(\theta'_A)) = \theta'_A$ implies

$$\frac{\partial u_A(\theta'_A; \theta_A)}{\partial \theta'_A} = -\dot{p}_B(\theta'_A) [\theta_A - \theta'_A] \mu_B(p_B(\theta'_A))$$

so that truthful reporting $\theta'_A = \theta_A$ satisfies the necessary condition for optimality, independently from the belief μ_B. It is also sufficient, because p_B non-increasing implies that

$$\theta'_A \geq \theta_A \Longrightarrow \frac{\partial u_A(\theta'_A; \theta_A)}{\partial \theta'_A} \leq 0,$$
$$\theta'_A \leq \theta_A \Longrightarrow \frac{\partial u_A(\theta'_A; \theta_A)}{\partial \theta'_A} \geq 0.$$

Similar characterization holds for users on side B. Finally, the inverse pricing mechanism is also ex post individually rational, because interactions occur only when interaction benefits exceed prices charged and outside options are normalized to zero. □

Observe that inverse pricing implies the following equivalence:

$$\theta_A = p_A(p_B(\theta_A)) \geq p_A(\theta_B) \Longleftrightarrow \theta_B = p_B(p_A(\theta_B)) \geq p_B(\theta_A) \tag{1}$$

This equivalence reduces the two-sided screening problem to a one-sided Mussa and Rosen (1978) problem. An interaction provides a positive net utility to one side if and only if this is true for the other side as well. Furthermore, users with a high interaction benefit are matched with more users on the other side of the market, because the interaction price charged from the other side is lower. Thus the lowest types on one side interact only with the highest types on the other side, whereas the highest types interact both with high and low types.

3 Results

The aim of the following sections is to derive the optimal second degree discriminatory platform pricing mechanisms for both profit and welfare maximization. The optimal mechanisms are inverse pricing mechanisms.

3.1 Profit maximization

Platform profit can be expressed as the total welfare less the user surplus on both sides of the market. By Lemma 1, an individual user on side A obtains the net utility

$$u_A(\theta_A) = \int_{p_B(\theta_A)}^{\overline{\theta}_B} [\theta_A - p_A(\theta_B)] f_B(\theta_B) \, d\theta_B$$

from joining the platform. This net utility is zero for the lowest type $\theta_A = p_A(\overline{\theta}_B)$ who makes at least one interaction on the platform, i.e. interacts at least with the highest type on the other side of the market. Integrating over all users on side A, we obtain the user surplus for that side of the market:

$$\begin{aligned} U_A &= \int_{\underline{\theta}_A}^{\overline{\theta}_A} \int_{p_B(\theta_A)}^{\overline{\theta}_B} [\theta_A - p_A(\theta_B)] f_B(\theta_B) f_A(\theta_A) \, d\theta_B d\theta_A \\ &= \int_{\underline{\theta}_B}^{\overline{\theta}_B} \left[\int_{p_A(\theta_B)}^{\overline{\theta}_A} [\theta_A - p_A(\theta_B)] f_A(\theta_A) \, d\theta_A \right] f_B(\theta_B) \, d\theta_B \end{aligned}$$

where the second equality follows from the equivalence 1 and Fubini's theorem. Using integration by parts in the brackets, we obtain

$$U_A = \int_{\underline{\theta}_B}^{\overline{\theta}_B} \int_{p_A(\theta_B)}^{\overline{\theta}_A} [1 - F_A(\theta_A)] f_B(\theta_B) \, d\theta_A d\theta_B$$

Similarly, the user surplus for side B writes

$$\begin{aligned} U_B &= \int_{\underline{\theta}_A}^{\overline{\theta}_A} \int_{p_B(\theta_A)}^{\overline{\theta}_B} [1 - F_B(\theta_B)] f_A(\theta_A) \, d\theta_B d\theta_A \\ &= \int_{\underline{\theta}_B}^{\overline{\theta}_B} \int_{p_A(\theta_B)}^{\overline{\theta}_A} [1 - F_B(\theta_B)] f_A(\theta_A) \, d\theta_B d\theta_A \end{aligned}$$

where the second equality again follows from Fubini's theorem and the equivalence in 1. Consequently, the total user surplus can be expressed as

$$U_A + U_B = \int_{\underline{\theta}_B}^{\overline{\theta}_B} \int_{p_A(\theta_B)}^{\overline{\theta}_A} \left[\frac{1 - F_A(\theta_A)}{f_A(\theta_A)} + \frac{1 - F_B(\theta_B)}{f_B(\theta_B)} \right] f_A(\theta_A) f_B(\theta_B) \, d\theta_A d\theta_B$$

The platform's profit equals the total welfare less the total user surplus:

$$\pi = \int_{\underline{\theta}_B}^{\overline{\theta}_B} \int_{p_A(\theta_B)}^{\overline{\theta}_A} \left[\theta_A + \theta_B - c - \frac{1 - F_A(\theta_A)}{f_A(\theta_A)} - \frac{1 - F_B(\theta_B)}{f_B(\theta_B)} \right] f_A(\theta_A) f_B(\theta_B) \, d\theta_A d\theta_B$$

Using the fact that

$$\int_{p_A(\theta_B)}^{\overline{\theta}_A} \left[\theta_A - \frac{1 - F_A(\theta_A)}{f_A(\theta_A)} \right] f_A(\theta_A) \, d\theta_A = p_A(\theta_B) \left[1 - F_A(p_A(\theta_B)) \right]$$

we obtain the following profit-maximization problem:

$$\max_{p_A(\cdot)} \int_{\underline{\theta}_B}^{\overline{\theta}_B} \left[p_A(\theta_B) + \theta_B - c - \frac{1 - F_B(\theta_B)}{f_B(\theta_B)} \right] \left[1 - F_A(p_A(\theta_B)) \right] f_B(\theta_B) \, d\theta_B$$

The platform chooses the profit-maximizing price schedule for side A users as a solution to to the first-order condition:

$$p_A(\theta_B) + \theta_B - \frac{1 - F_B(\theta_B)}{f_B(\theta_B)} - c = \frac{1 - F_A(p_A(\theta_B))}{f_A(p_A(\theta_B))}$$

Observe that by plugging in the marginal side B user we obtain the optimal price $p_B(\theta_A)$ schedule for side B. Proposition 2 casts the first-order conditions in terms of matching elasticities and virtual interaction benefits, and shows that the platform cannot do any better by choosing some other mechanism.

Proposition 2. *The profit-maximizing pricing mechanism is the inverse pricing mechanism that satisfies the Lerner formulae*

$$\frac{p_A(\theta_B) - [c - b_B(\theta_B)]}{p_A(\theta_B)} = \frac{1}{\eta_A(\theta_B)}$$
$$\frac{p_B(\theta_A) - [c - b_A(\theta_A)]}{p_B(\theta_A)} = \frac{1}{\eta_B(\theta_A)}$$

Proof. The Lerner formulae follows directly from the first-order conditions, which are also sufficient and satisfy the monotonicity constraint due to Assumption 1. Furthermore, since virtual interaction benefits are non-decreasing, the platform's value function associated with the inverse pricing mechanism can be written as

$$\int_{\underline{\theta}_A}^{\overline{\theta}_A} \int_{\underline{\theta}_B}^{\overline{\theta}_B} \left[\theta_A + \theta_B - c - \frac{1 - F_A(\theta_A)}{f_A(\theta_A)} - \frac{1 - F_B(\theta_B)}{f_B(\theta_B)} \right] m(\theta_A, \theta_B) f_A(\theta_A) f_B(\theta_B) \, d\theta_B d\theta_A$$

where

$$m(\theta_A, \theta_B) = \begin{cases} 1 & \text{if } \theta_A + \theta_B - c \geq \frac{1-F_A(\theta_A)}{f_A(\theta_A)} + \frac{1-F_B(\theta_B)}{f_B(\theta_B)} \\ 0 & \text{otherwise} \end{cases}$$

Let us now show that the platform cannot find another incentive compatible and individually rational mechanism that yields a higher profit. A mechanism is incentive compatible for side A if and only if N_B is non-decreasing in θ_A and we can write (Myerson, 1978):

$$u_A(\theta_A) = u_A(\underline{\theta}_A) + \int_{\underline{\theta}_A}^{\theta_A} N_B(x_A)\, dx_A$$

The total user surplus for side A is then

$$\begin{aligned} U_A = \int_{\underline{\theta}_A}^{\overline{\theta}_A} u_A(\theta_A) f_A(\theta_A)\, d\theta_A &= u_A(\underline{\theta}_A) + \int_{\underline{\theta}_A}^{\overline{\theta}_A} \int_{\underline{\theta}_A}^{\theta_A} N_B(x_A)\, dx f_A(\theta_A)\, d\theta_A \\ &= u_A(\underline{\theta}_A) + \int_{\underline{\theta}_A}^{\overline{\theta}_A} N_B(\theta_A) \frac{1-F_A(\theta_A)}{f_A(\theta_A)} f_A(\theta_A)\, d\theta_A \end{aligned}$$

where the second equality follows from Fubini's theorem and $u_A(\underline{\theta}_A) \geq 0$ is required by individual rationality. Similarly, the mechanism is incentive compatible for side B if and only if N_A is non-decreasing in θ_B and

$$U_B = u_B(\underline{\theta}_B) + \int_{\underline{\theta}_B}^{\overline{\theta}_B} N_A(\theta_B) \frac{1-F_B(\theta_B)}{f_B(\theta_B)} f_B(\theta_B)\, d\theta_B$$

where $u_B(\underline{\theta}_B) \geq 0$. Letting $q : \Theta \longrightarrow [0,1]$ denote an arbitrary matching probability, define

$$\begin{aligned} N_A(\theta_B) &\equiv \int_{\underline{\theta}_A}^{\overline{\theta}_A} q(\theta_A, \theta_B) f_A(\theta_A)\, d\theta_A \\ N_B(\theta_A) &\equiv \int_{\underline{\theta}_B}^{\overline{\theta}_B} q(\theta_A, \theta_B) f_B(\theta_B)\, d\theta_B \end{aligned}$$

Consequently,

$$U_A + U_B = u_A(\underline{\theta}_A) + u_B(\underline{\theta}_B) + \int_{\underline{\theta}_A}^{\overline{\theta}_A} \int_{\underline{\theta}_B}^{\overline{\theta}_B} \left[\frac{1-F_A(\theta_A)}{f_A(\theta_A)} + \frac{1-F_B(\theta_B)}{f_B(\theta_B)}\right] q(\theta_A, \theta_B) f_A(\theta_A) f_B(\theta_B)\, d\theta_A d\theta_B$$

The total welfare writes:

$$\begin{aligned} W = &\int_{\underline{\theta}_A}^{\overline{\theta}_A} \theta_A N_B(\theta_A) f_A(\theta_A)\, d\theta_A + \int_{\underline{\theta}_B}^{\overline{\theta}_B} \theta_B N_A(\theta_B) f_B(\theta_B)\, d\theta_B \\ &- \int_{\underline{\theta}_A}^{\overline{\theta}_A} \int_{\underline{\theta}_B}^{\overline{\theta}_B} c q(\theta_A, \theta_B) f_A(\theta_A) f_B(\theta_B)\, d\theta_A d\theta_B \\ &= \int_{\underline{\theta}_A}^{\overline{\theta}_A} \int_{\underline{\theta}_B}^{\overline{\theta}_B} [\theta_A + \theta_B - c]\, q(\theta_A, \theta_B) f_A(\theta_A) f_B(\theta_B)\, d\theta_A d\theta_B \end{aligned}$$

Platform profit is the total welfare less the total user surplus:

$$\pi = \int_{\underline{\theta}_A}^{\overline{\theta}_A} \int_{\underline{\theta}_B}^{\overline{\theta}_B} \left[\theta_A + \theta_B - c - \frac{1 - F_A(\theta_A)}{f_A(\theta_A)} - \frac{1 - F_B(\theta_B)}{f_B(\theta_B)} \right] q(\theta_A, \theta_B) f_A(\theta_A) f_B(\theta_B) d\theta_A d\theta_B$$
$$- u_A(\underline{\theta}_A) - u_B(\underline{\theta}_B)$$

By construction, $q = m$ together with

$$u_A(\underline{\theta}_A) = u_B(\underline{\theta}_B) = 0$$

maximizes the platform's profit. □

It is useful to compare the profit-maximizing pricing mechanism to the benchmark mechanism. Take an individual user on side B with an interaction benefit. Now the difference to linear pricing is that the optimal monopoly price charged from side A depends on the user's interaction benefit. This price is chosen based on the cost of interaction less the virtual interaction benefit for the user on side B, which equals her true interaction benefit less the cost of information. Importantly, optimal pricing satisfies a *perfect* see-saw condition: given an interaction, if the price charged from one side is decreased, then the price for the other side is increased one-to-one. This is the key property of inverse pricing, translating the two-sided pricing problem into a one-sided screening problem. This implies that the cost pass-throughs are independent and equal to the pass-through rates:

$$\frac{dp_A(\theta_B)}{dc} = \rho_A(\theta_B)$$
$$\frac{dp_B(\theta_A)}{dc} = \rho_B(\theta_A)$$

By contrast, under linear pricing, the price changes resulting from an increase in the cost of intermediation depend on pass-through rates on both sides:

$$\frac{dp_A}{dc} = \frac{\rho_A(1 - \rho_B)}{1 - \rho_A \rho_B}$$
$$\frac{dp_B}{dc} = \frac{\rho_B(1 - \rho_A)}{1 - \rho_A \rho_B}$$

No distortion at the top Proposition 2 shows that there is no distortion at the top: the monopoly price charged from side A for interactions with the highest user type on side B equals the one-sided monopoly price, i.e. the virtual interaction benefit equals the true interaction benefit:

$$\frac{p_A(\overline{\theta}_B) - (c - \overline{\theta}_B)}{p_A(\overline{\theta}_B)} = \frac{1}{\eta_A(\overline{\theta}_B)}$$

In particular, this monopoly price is the lowest price charged from the users on side A. Thus the user with the highest interaction benefit has an incentive to report her type truthfully in order to maximize the number of matches with the other side of the market. In essence, the number of matches determines the quality of the platform for a given user, hence the resemblance to the canonical screening model of Mussa and Rosen (1978) for one-sided markets.

3.2 Welfare maximization

Welfare from a side B user with an interaction benefit θ_B can be written in two parts. First, the joint profit between the user and the platform. Second, the surplus to side A users who interact with the user. Integrating then over all users on side B yields the following expression for total welfare:

$$W = \int_{\underline{\theta}_B}^{\bar{\theta}_B} \left[\left(p_A\left(\theta_B\right) + \theta_B - c\right) \left[1 - F_A\left(p_A\left(\theta_B\right)\right)\right] + \int_{p_A(\theta_B)}^{\bar{\theta}_A} \left[1 - F_A\left(\theta_A\right)\right] d\theta_A \right] f_B\left(\theta_B\right) d\theta_B$$

The price schedule that maximizes welfare is the Groves mechanism:

$$p_A\left(\theta_B\right) = c - \theta_B \quad \text{and} \quad p_B\left(\theta_A\right) = c - \theta_A$$

This welfare maximizing pricing mechanism belongs to the class of inverse pricing mechanisms, because both the monotonicity constraint and the inverse relation in Definition 1 are satisfied. Furthermore, the Pigouvian condition is satisfied: the price charged from one side equals the cost of interaction less the "external" interaction benefit obtained by the user on the other side of the market. If the interaction benefit on the other side is positive, the interaction should be subsidized and taxed otherwise. This implies that the mechanism is generally not budget balanced for the platform, who must incentivize users on both sides to make them internalize the total welfare from interactions. The platform makes a loss equal to

$$\begin{aligned}
\pi &= \int_{\underline{\theta}_B}^{\bar{\theta}_B} \left[p_A\left(\theta_B\right) + \theta_B - c - \frac{1 - F_B\left(\theta_B\right)}{f_B\left(\theta_B\right)} \right] \left[1 - F_A\left(p_A\left(\theta_B\right)\right)\right] f_B\left(\theta_B\right) d\theta_B \\
&= - \int_{\underline{\theta}_B}^{\bar{\theta}_B} \left[1 - F_B\left(\theta_B\right)\right] \left[1 - F_A\left(p_A\left(\theta_B\right)\right)\right] d\theta_B \leq 0
\end{aligned}$$

To avoid this loss, let us consider welfare maximization

$$\max_{p_A(\cdot)} \int_{\underline{\theta}_B}^{\bar{\theta}_B} \left[\left(p_A\left(\theta_B\right) + \theta_B - c\right) \left[1 - F_A\left(p_A\left(\theta_B\right)\right)\right] + \int_{p_A(\theta_B)}^{\bar{\theta}_A} \left[1 - F_A\left(\theta_A\right)\right] d\theta_A \right] f_B\left(\theta_B\right) d\theta_B$$

subject to budget balance: $\pi \geq 0$. Proposition 3 summarizes the optimal pricing mechanism.

Proposition 3. *The budget balanced welfare-maximizing pricing mechanism is the inverse pricing mechanism that satisfies the Lerner formulae*

$$\begin{aligned}
\frac{p_A\left(\theta_B\right) - \left[c - \theta_B\right]}{p_A\left(\theta_B\right)} + \lambda \frac{p_A\left(\theta_B\right) - \left[c - b_B\left(\theta_B\right)\right]}{p_A\left(\theta_B\right)} &= \lambda \frac{1}{\eta_A\left(\theta_B\right)} \\
\frac{p_B\left(\theta_A\right) - \left[c - \theta_A\right]}{p_B\left(\theta_A\right)} + \lambda \frac{p_B\left(\theta_A\right) - \left[c - b_A\left(\theta_A\right)\right]}{p_B\left(\theta_A\right)} &= \lambda \frac{1}{\eta_B\left(\theta_A\right)}
\end{aligned}$$

where λ is positive and solves the budget constraint $\pi = 0$.

Proof. Denoting $\lambda \geq 0$ as the multiplier for the budget constraint $\pi \geq 0$, we have $\lambda\pi = 0$ and the first-order condition writes

$$p_A(\theta_B) + \theta_B - c = \frac{\lambda}{1+\lambda}\frac{1-F_A(p_A(\theta_B))}{f_A(p_A(\theta_B))} + \frac{\lambda}{1+\lambda}\frac{1-F_B(\theta_B)}{f_B(\theta_B)}$$

which is sufficient and satisfies Definition 1 for inverse pricing due to Assumption 1. Suppose first that $\pi > 0 \implies \lambda = 0$. Then we have Pigouvian prices, which imply $\pi \le 0$, a contradiction. Thus $\pi = 0$ must hold, which implies

$$\lambda = \frac{\int_{\underline{\theta}_B}^{\overline{\theta}_B} \frac{1-F_B(\theta_B)}{f_B(\theta_B)}\left[1-F_A(p_A(\theta_B))\right] f_B(\theta_B)\, d\theta_B}{\int_{\underline{\theta}_B}^{\overline{\theta}_B} \frac{1-F_A(p_A(\theta_B))}{f_A(p_A(\theta_B))}\left[1-F_A(p_A(\theta_B))\right] f_B(\theta_B)\, d\theta_B}$$

Using the definitions for the matching elasticities we obtain the elasticity rules in the result. The value of the Lagrangian function associated with the optimal inverse pricing mechanism can be written as

$$\int_{\underline{\theta}_A}^{\overline{\theta}_A}\int_{\underline{\theta}_B}^{\overline{\theta}_B}\left[\theta_A + \theta_B - c - \frac{\lambda}{1+\lambda}\frac{1-F_A(\theta_A)}{f_A(\theta_A)} - \frac{\lambda}{1+\lambda}\frac{1-F_B(\theta_B)}{f_B(\theta_B)}\right] m(\theta_A,\theta_B) f_A(\theta_A) f_B(\theta_B)\, d\theta_B d\theta_A$$

where

$$m(\theta_A,\theta_B) = \begin{cases} 1 & \text{if } \theta_A + \theta_B - c \ge \frac{\lambda}{1+\lambda}\frac{1-F_A(\theta_A)}{f_A(\theta_A)} + \frac{\lambda}{1+\lambda}\frac{1-F_B(\theta_B)}{f_B(\theta_B)} \\ 0 & \text{otherwise} \end{cases}$$

Let us now show that there is no other incentive compatible and individually rational mechanism that yields a higher welfare, subject to the budget constraint. Following the same steps as in the proof of Proposition 2, we obtain the welfare

$$W = \int_{\underline{\theta}_A}^{\overline{\theta}_A}\int_{\underline{\theta}_B}^{\overline{\theta}_B}\left[\theta_A + \theta_B - c\right] q(\theta_A,\theta_B) f_A(\theta_A) f_B(\theta_B)\, d\theta_A d\theta_B$$

and platform profit

$$\begin{aligned}\pi = &\int_{\underline{\theta}_A}^{\overline{\theta}_A}\int_{\underline{\theta}_B}^{\overline{\theta}_B}\left[\theta_A + \theta_B - c - \frac{1-F_A(\theta_A)}{f_A(\theta_A)} - \frac{1-F_B(\theta_B)}{f_B(\theta_B)}\right] q(\theta_A,\theta_B) f_A(\theta_A) f_B(\theta_B)\, d\theta_A d\theta_B \\ &- u_A(\underline{\theta}_A) - u_B(\underline{\theta}_B)\end{aligned}$$

Denoting $\nu \ge 0$ as the multiplied for the constraint $\pi \ge 0$, the Lagrangian writes

$$\begin{aligned}L = &\int_{\underline{\theta}_A}^{\overline{\theta}_A}\int_{\underline{\theta}_B}^{\overline{\theta}_B}\left[(1+\nu)\left[\theta_A + \theta_B -\right] c - \nu\frac{1-F_A(\theta_A)}{f_A(\theta_A)} - \nu\frac{1-F_B(\theta_B)}{f_B(\theta_B)}\right] q(\theta_A,\theta_B) f_A(\theta_A) f_B(\theta_B)\, d\theta_A d\theta_B \\ &- \nu u_A(\underline{\theta}_A) - \nu u_B(\underline{\theta}_B)\end{aligned}$$

By construction, this is maximized at

$$u_A(\underline{\theta}_A) + u_B(\underline{\theta}_B) = 0$$

and $q = m$, $\nu = \lambda$. □

Proposition 3 shows that the budget balanced welfare-maximizing pricing mechanism is an inverse pricing mechanism that weights Pigouvian pricing and profit maximization to satisfy the budget constraint: Pigouvian pricing is attained at $\lambda \longrightarrow 0$ and profit maximization at $\lambda \longrightarrow \infty$. Information is costly for the platform, which is the reason why the budget constraint at the first best is not satisfied in general. Yet optimal pricing still satisfies the perfect see-saw condition, so that a price decrease on one side triggers a one-to-one increase in price on the other side. Furthermore, for the highest interaction benefits there is no information distortion at the top: for the highest user type on side B, the price charged from side A satisfies:

$$(1+\lambda)\frac{p_A\left(\overline{\theta}_B\right)-\left(c-\overline{\theta}_B\right)}{p_A\left(\overline{\theta}_B\right)}=\lambda\frac{1}{\eta_A\left(\overline{\theta}_B\right)}$$

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